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# STUDENTS' STRATEGIES IN SOLVING ALGORITHMIC STOICHIOMETRY PROBLEMS 

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#### Abstract

Students solving chemistry problems tend to rely on algorithmic methods. However, the problem solving strategy a person applies may depend on the difficulty of the problem. Research on stoichiometric problem solving reports that students working on easy-to-calculate problems created a "non-mathematical" strategy on their own. The aim of the present study was to find out (1) whether Swedish students would behave in a similar manner to that described in literature and (2) how they change the strategies when moving from an easy-to-calculate problem to a more difficult question. Four semistructured interviews were conducted with Swedish senior high school students. The result showed that they reacted as described in literature. In addition to applying mathematical strategies they each developed a non-mathematical strategy. All the students used a non- mathematical strategy to solve an easy question. However, in moving from an easy-to-calculate problem to a more difficult one most students calculated the mass fraction or the percentages of an element in a compound. In this form the strategy comes close to the non-mathematical strategy. It is suggested for introductory chemistry courses to use easy-to-calculate problems and to concentrate on both the non-mathematical and the mass fraction strategy. [Chem. Educ. Res. Pract.: 2003, 4, 305-317]


KEY WORDS : stoichiometry; problem solving strategy; problem difficulty; easy problems; semistructured interview

## INTRODUCTION

Research reports that when solving chemistry problems many students tend to use algorithmic methods. This is especially true for students who have not sufficiently grasped the chemistry behind a problem. They may use a memorized formula, manipulate the formula and plug in numbers until they fit etc. (Gabel \& Bunce, 1994). Teaching reinforces this behaviour if it emphasizes correct answers rather than students' understanding of chemistry (Nakhleh \& Mitchell, 1993).

Schmidt (1994) put forward the hypothesis that the problem-solving strategy a person applies depends on the difficulty of the problem (p. 198). A difficult problem is preferably solved using algorithmic methods students have learned. If a problem is easy, the person is inclined to use a strategy based on reasoning. This hypothesis was used to explain the finding that when solving easy problems in stoichiometry, German high school students invented their own strategy. Working on more difficult problems, students relied on what they knew and used a mathematical strategy, for example a method suggested by textbooks and/or teachers.

This study investigates Swedish senior high school students` strategies for solving stoichiometry problems.

## BACKGROUND

Stoichiometry is the branch of chemistry evaluating the results of quantitative measurements connected to chemical compounds and reactions. Sample questions to be answered through stoichiometric calculations are:
(1) How much iron can be obtained from one ton of iron ore?
(2) How much nitrogen and hydrogen are required to make one ton of ammonia?

Stoichiometric calculations are also needed to evaluate the results of quantitative analyses like titrations. Chemical formulae can be calculated if it is known how much of each element is present in a compound. For these reasons stoichiometry has become an important topic in curricula and chemistry textbooks, and many investigations have been carried out to understand students' problems in this field. Those published papers on stoichiometric problem solving that have been reviewed are Griffiths (1994), Gabel and Bunce (1994) as well as Furio et al. (2002).

Stoichiometric calculation belongs to the less attractive and more difficult areas in high school chemistry. Matriculation results in Israel have shown that achievement on mole concept questions was lower than the average achievement in the examination as a whole (Novick \& Menis, 1976). Gable and Sherwood (1984) administered two tests to senior high school students. One problem set consisted of mole concept tasks, the other of analogue tasks. The analogue tasks were about oranges and other market produce which replaced chemical terms like sodium hydroxide, and concepts such as dozens instead of moles. Many students who successfully solved the analogue tasks were unable to solve the mole concept tasks. It seemed that the difficulties students had in solving the mole concept tasks were not due to the arithmetic needed but to the lack of understanding chemical terms including the mole. Schmidt (1994) studied senior high school students' strategies for solving stoichiometry problems. He also proposed some teaching ideas to introduce stoichiometry (Schmidt, 1997). Dori and Hameri (1996) developed a learning programme for stoichiometry containing problems of varying difficulty. They also developed and validated an analysis system to predict the rate of students' success in solving mole-related problems (Dori \& Hameri, 2003).

The original Nuffield curriculum introduced stoichiometric calculations already in junior high schools. "The quantitative aspect in chemical investigation is of fundamental importance. It must be present from the beginning" (Nuffield Foundation, 1967, p. 68). In Sweden, stoichiometry is introduced at senior high school level (Andersson et al., 2000). Swedish textbooks present a variety of problems in stoichiometry. They differ in what is given and what is required: mass, gas volume, amount of substance, molar mass, percent composition, volume of solution, molar concentration, and mass concentration. Item 1 is among the more easy-to-calculate problems which are based on binary compounds (Andersson et al., 2000, p. 72).

## Item 1

What mass of silver would be found in 0.1346 g silver chloride, AgCl ?
This example is used to illustrate how a problem of this type can be solved. Students are advised first to determine the molar mass of silver chloride and the mass percentage of
silver. Multiplying the given mass of 0.1346 g for silver chloride by the mass percentage of silver gives 0.1013 g silver. To do the arithmetic for this type of problem, a calculator is needed.

Multiple choice items have been developed that can easily be solved without arithmetical calculations and without a calculator (Schmidt \& Beine 1992; Schmidt 1994; Schmidt 1997). To make this possible, the relative atomic masses (or molar masses) of the elements are rounded off to whole numbers. The problems are based on binary compounds the elements in which have a mass ratio of $1: 1$ and a simple atomic (molar) mass ratio such as $2: 1,3: 1$. The rounded atomic (molar) masses are given in brackets below.

Li (7), N (14)
C (12), Mg (24), Cl (36), Ti (48)
O (16), S (32), Ti (48), Cu (64), Mo (96), Te (128)
Item 2 illustrates problems of this type. It refers to copper sulfide. In this case the atomic (molar) mass ratio $\mathrm{M}(\mathrm{Cu}): \mathrm{M}(\mathrm{S})$ is $64 \mathrm{~g} / \mathrm{mol}: 32 \mathrm{~g} / \mathrm{mol}=2: 1$, the mass ratio $\mathrm{m}(\mathrm{Cu}): \mathrm{m}(\mathrm{S})$ $=1: 1$.

## Item 2

The chemical formula of copper sulfide is $\mathrm{CuS}_{2}$. What mass of sulfur would be found in 6 g of copper sulfide?

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[A] 1.5g,[B] 2g,[C] 3g,[D] 4g
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Schmidt $(1994,1997)$ administered items of this type to several thousand senior high school students. He observed that students used three strategies to solve the problems. These are illustrated with reference to item 2 :

## Strategy 1 (mole method):

1. Calculate the molar mass of the compound
$M\left(\mathrm{CuS}_{2}\right)=128 \mathrm{~g} / \mathrm{mol}$
2. Calculate the amount of substance of the compound
$n\left(\mathrm{CuS}_{2}\right)=6 \mathrm{~g} /(128 \mathrm{~g} / \mathrm{mol})=(6 / 128) \mathrm{mol}$.
3. Determine the amount of substance of the element whose mass is requested

1 mol CuS ${ }_{2}$ contains 2 moles $S$ $n(S)=(12 / 128) \mathrm{mol} S$.
4. Calculate the mass of sulfur $m(S)$ $m(S)=n(S) * M(S)=(12 / 128) \mathrm{mol} * 32 \mathrm{~g} / \mathrm{mol} S=3 \mathrm{~g} \mathrm{~S}$

Typical of strategy 1 are steps 2 and 3 describing the relations between given and required substance via amount of substance.

## Strategy 2 (proportionality method):

1. Calculate the molar mass of the compound $M\left(\mathrm{CuS}_{2}\right)=128 \mathrm{~g} / \mathrm{mol}$
2. Formulate the ratio between masses and molar masses (one mole $\mathrm{CuS}_{2}$ consists of one mole Cu and two moles of S):
$\frac{m(S)}{m\left(\mathrm{CuS}_{2}\right)}=\frac{2 M(\mathrm{~S})}{M\left(\mathrm{CuS}_{2}\right)}$
3. Transform the ratio to
$m(S)=\ldots$
4. Calculate the mass requested

$$
m(S)=\frac{2 * 32 g / m o l}{128 g / m o l} * 6 g C u=3 g C u
$$

Typical of this strategy are steps 2 and 3, forming a relation between given and required substances via a proportion. It is, in fact, the proportionality method that the Swedish textbook mentioned earlier (Andersson et al., 2000) recommends for solving item 1 (p. 72). In step 1 the molar mass of silver chloride is determined. Step 2 refers to the calculation of the mass percentage of silver in the compound. This corresponds to the right side of the equation. In steps 3 and 4 the mass percentage of silver is multiplied with the given mass of silver chloride. Students from Schmidt's study solved the more difficult item 3 (mass lithium in 60 g of lithium nitride) in this way also; they calculated the "portion of lithium" (Schmidt 1997, p. 246).

## Strategy 3 ("logical method")

1. The molar (atomic) mass ratio of copper to sulfur $M(C u): M(S)$ is $2: 1$
i.e. two atoms of sulfur have the same mass as one atom of copper.
2. The atom ratio of copper to sulfur $n(C u): n(S)$ is conversely $1: 2$
3.Consequently the mass ratio of copper to sulfur, $m(C u): m(S)$, is $1: .1$
3. Dividing the 6 g of copper sulfide into two halves leads to 3 g sulfur.

It is typical of this strategy that students described the relations between variables in their own words, for example "twice as much", "same proportion", etc. instead of applying mathematical algorithms. They made comments like: "There is no need to calculate here, the problem can be solved by pure logic" (Schmidt, 1997). Strategy 3, however, is not mentioned in any German textbook.

Students frequently chose strategy 3 for problems that were based on chemical compounds with a mass ratio of its elements $1: 1$. But when the mass ratio deviated from 1 : 1 as in item 3 fewer students used strategy 3 to solve the problem.

## Item 3

The chemical formula of lithium nitride is $\mathrm{Li}_{3} \mathrm{~N}$. What mass of lithium would be found in 60 g of lithium nitride?
[A] $12 \mathrm{~g},[B] \quad 20 \mathrm{~g},[\mathrm{C}] 36 \mathrm{~g},[\mathrm{D}] \quad 45 \mathrm{~g}$
Strategies 2 and 3 "both avoid direct calculation of amounts in moles, which is... necessary in" strategy 1. "It is open to discussion as to whether this happens because students encountered difficulties with the mole concept" (Schmidt, 1994).

Schmidt $(1994,1997)$ also reported that students had difficulties piecing together the atom ratio, mass ratio and the molar mass ratio. A frequent error was to equate the mass ratio with the mol (atom) ratio. For item 2 this would mean:

$$
\mathrm{m}(\mathrm{Cu}): \mathrm{m}(\mathrm{~S})=\mathrm{n}(\mathrm{Cu}): \mathrm{n}(\mathrm{~S})=1: 2
$$

Students dividing the 6 g copper sulfide in the relation $1: 2$ would be led to the incorrect result of 4 g of sulfur.

Another (less frequent) error was to equate the mass ratio with the ratio of molar masses. For item 2 this would mean

$$
\mathrm{m}(\mathrm{Cu}): \mathrm{m}(\mathrm{~S})=\mathrm{M}(\mathrm{Cu}): \mathrm{M}(\mathrm{~S})=2: 1
$$

Students dividing the 6 g copper sulfide in the relation 2:1 would arrive at another incorrect result 2 g sulfur.

At this point the reason for using 6 g of copper sulfide in item 2 becomes obvious. The number 6 can easily be divided into the ratios $n(C u): n(S)=1: 2, M(C u): M(S)=2: 1$, and $\mathrm{m}(\mathrm{Cu}): \mathrm{m}(\mathrm{S})=1: 1$. There are similar reasons for using other numbers for the given mass in problems that refer to the formulae of other compounds (see Table 1).

## AIMS OF THE STUDY

In the laboratory, students are confronted with problems like item 1. Here it is not easy to apply strategy 3 . Students may adopt an arithmetical strategy. At this point they are tempted to learn a strategy by heart and make it an algorithm. Schmidt (1997) recommends introducing the basic principles of stoichiometric calculation using problems like item 2. Working on these problems, students will reinvent the logical strategy. They may be asked to describe their strategy in their own words. From Schmidt's study, it can be expected that if students are given more difficult problems like item 3 to solve, strategies 1 and 2 will be preferred. Knowing about the difficulties students may have solving items 2 and 3 could aid us in our decision as to which mathematical strategy to focus on in our teaching. Schmidt collected his data through written tests. Therefore, we do not know how students solving item 2 first and then item 3 move from strategy 3 to strategy 1 or 2 . At the moment we even do not know whether or not students in Sweden would solve the stoichiometry problems discussed here in the same ways as German students. These considerations lead to the following purposes of the study, which were to find out:
(1) how Swedish senior high school students solve problems like item 2, and whether they develop the "logical strategy" (strategy 3), and
(2) how the individual student changes his or her strategy when first solving a problem like item 2 followed by a more difficult problem like item 3 .

## METHOD

## Data collection and sample

To answer the research questions interviews were planned. Swedish textbooks introduce stoichiometry in the first year of senior high school. We were interested in studying the strategies of students who had already practiced stoichiometric calculations. It was, therefore, decided to focus on second year (grade 12) students from senior high school. Six students (five girls and one boy) were selected from a science course of a larger urban school in central Sweden. They volunteered for the interviews (convenience sample). The students had been introduced to stoichiometric calculation the previous year. The interviews lasted between 20 and 30 minutes and were conducted in a group room of the school by one of the
authors (C. J.), who taught in that class as part of her teaching training. The tape recordings were transcribed for later analysis.

## Design of the interviews

The interviews were planned according to Kvale (1996). They consisted of the briefing and the warming-up phases followed by the introduction and the main phase, and were concluded by the debriefing. In the briefing the interviewer presented some ideas about why chemistry education research is needed. The procedure of the interviews was also explained. Students were informed of their right to withdraw from the interview at any time. In the warming-up phase some general questions about whether the student liked or disliked chemistry were asked. In the debriefing the chemistry content of the interview was discussed. The interviewees were also given the chance to ask the interviewer questions of any kind.

In the introductory phase students were shown a sheet of paper with the chemical formula $\mathrm{SO}_{2}$ on it. Students were asked questions from an interview guide to probe their understanding of a chemical formula. They were also provided with a list of the molar/atomic masses (rounded off) of the elements $\mathrm{H}, \mathrm{Li}, \mathrm{C}, \mathrm{N}, \mathrm{O}, \mathrm{Mg}, \mathrm{S}, \mathrm{Cl}, \mathrm{Ti}, \mathrm{Cu}, \mathrm{Mo}, \mathrm{Te}$. In this list no units were given, thus offering students a choice of referring to molar or relative atomic masses in their solutions.

In the main phase of the interviews students were asked to solve items 2,3 , and 5. Item 4 was only used in the probational interview. The items were developed according to Schmidt (1997) and had the following characteristics (Table 1):

TABLE 1: Data for the construction of the test items used in the interviews.

| Item | formula | $\mathbf{n}_{\mathbf{1}}: \mathbf{n}_{\mathbf{2}}$ | $\mathbf{M}_{\mathbf{1}}: \mathbf{M}_{\mathbf{2}}$ | $\mathbf{m}_{\mathbf{1}}: \mathbf{m}_{\mathbf{2}}$ | mass of the compound |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2}$ | $\mathrm{CuS}_{2}$ | $1: 2$ | $2: 1$ | $1: 1$ | 6 g |
| $\mathbf{3}$ | $\mathrm{Li}_{3} \mathrm{~N}$ | $3: 1$ | $1: 2$ | $3: 2$ | 60 g |
| $\mathbf{4}$ | $\mathrm{MoS}_{3}$ | $1: 3$ | $3: 1$ | $1: 1$ | 4 g |
| $\mathbf{5}$ | $\mathrm{TiO}_{2}$ | $1: 2$ | $3: 1$ | $3: 2$ | 60 g |

From the mass ratio of the elements it can be seen that items 2 and 4 belong to the more easy-to-solve questions, items 3 and 5 to the more difficult ones.

## Item 4

The chemical formula of molybdenum sulfide is $\mathrm{MoS}_{3}$. What mass of sulfur would be found in 4 g of molybdenum sulfide?

$$
[A] \quad 0.66 \mathrm{~g},[B] \quad 1 \mathrm{~g},[\mathrm{C}] \quad 2 \mathrm{~g},[D] \quad 3 \mathrm{~g}
$$

## Item 5

The chemical formula of titanium dioxide is $\mathrm{TiO}_{2}$. What mass of oxygen would be found in 60 $g$ of titanium dioxide?
[A] 12 g, [B] $15 \mathrm{~g},[C] 24 \mathrm{~g},[D] 40 \mathrm{~g}$
Students were provided with a pen, a sheet of paper, and a pocket calculator.
The introductory and the main phase of the interviews were tape-recorded. After the interviews were conducted, the interviewer took notes concerning aspects the tape-recorder could not document, such as the atmosphere during the interviews and the students' behaviour.

The first interview was used to improve the interview guideline and to develop the interviewer's craftsmanship in conducting interviews. This interview is not included in the analysis.

## RESULTS AND INTERPRETATION

## The interviewees

One interview had to be terminated prior to its completion because the interviewee seemed to be stressed. The report that follows is, therefore, based on four interviews conducted with three girls and one boy (student 1 to 4 ).

For student 1 chemistry was one subject among others. During the interview she showed self-confidence in doing the calculations. Student 2 had a timid personality. The conversation was conducted in a quiet voice. She was eager to understand the calculations. Student 3 could hardly wait for the tasks to be given. When the problems were finally presented, she tackled them with great enthusiasm. She was confident about her knowledge in chemistry but sought confirmation. Student 4 showed confidence in doing the calculations. He needed some time to solve the problems, but did the calculations carefully and confirmed the results.

## The interviews

From the transcripts students' strategies for solving the problems were extracted. All quotes are translations from Swedish.

## Introductory phase

All students solved the problem "What mass of oxygen would you find in $6 \mathrm{~g} \mathrm{SO}_{2}$ ?" correctly using strategy 3 .

## Main phase item 2

Item 2 was similar to the introductory problem in its formulation and level of difficulty. All students solved this item correctly using strategy 3 . They gave the correct answer quickly and were confident about the logic of their arguments. Here is an example how student 2 reasoned:

S2: ... the same procedure as before, one sulfur weighs... $32 u, 2$ sulfur weigh $64 u$, and copper weighs... $64 u$, so the relation is $50: 50$. The answer, therefore, is 3 g .

Students first determined the molar mass ratio of the elements and, taking into account the atom ratio, calculated the mass ratio. Finally, the given mass of $6 \mathrm{~g} \mathrm{SO}_{2}$ was divided into two halves. Thus all steps of the logical strategy 3 are represented.

## Main phase item 4

This item was only used in the probational interview. The student arrived at the correct answer using strategy 3.

## Main phase item 3

All students solved this problem correctly, student 2 used strategy 1, students 1 and 3 strategy 2 , and student 4 strategy 3 .

TABLE 2: Strategies students used to solve the items.

| Student | item \# strategy | item \# strategy | item \# strategy |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $2 \# 3$ | $3 \# 2$ | $5 \# 2$ |
| $\mathbf{2}$ | $2 \# 3$ | $3 \# 1$ | $5 \# 1$ |
| $\mathbf{3}$ | $2 \# 3$ | $3 \# 2$ | $5 \# 2$ |
| $\mathbf{4}$ | $2 \# 3$ | $3 \# 3$ | $5 \# 3$ |

Student 1 who had solved item 2 using strategy 3 seemed unaware of this possibility for item 3. She directly turned to the mathematical strategy 2 :

## Step 1 (calculating the molar masses)

S1: ...because the atomic mass of lithium is 21 u...they have together a mass of 35 .
Step 2 (formulate the molar mass ratio $\mathbf{M}(\mathbf{L i}) / \mathbf{M}\left(\mathrm{Li}_{3} \mathbf{N}\right)$
S1: ... so I divide 21 by 35 to get the percentage. Then I multiply it with...
$\mathrm{I}: \quad$ What is the percentage?
S1: ... It is 0.6.

## Step 3/4 (calculating the mass required)

S1: ... and then I multiply the mass of 60 with...
I: $\quad 0.6$ times 60 gives 36 ?
S1: Yes, right.
This strategy deviates slightly from strategy 2 (see above) as the student calculated the percentage instead of referring to the proportions $\mathrm{M}(\mathrm{Li}) / \mathrm{M}\left(\mathrm{Li}_{3} \mathrm{~N}\right)=\mathrm{m}(\mathrm{Li}) / \mathrm{m}\left(\mathrm{Li}_{3} \mathrm{~N}\right)$. The basic idea, however, is the same.

Student 2 applied strategy 1.
Steps 1 and 2 (calculating the amount of substance of the compound)
S2: ... I divide 60 by 35, the result is 1.71 (writes it down and calculates by herself)

## Intermediate step (playing with numbers)

S2: ... this is multiplied by 21. The result is around 36 ...I must calculate what this is in per cent... no, I don't need...I don't know how to explain it. But if I divide the total mass by the mass for one...so I'll get... how much... 1 u is in gram.

Student 2 was not aware that she started calculating the amount of substance. It occurred to her to move on with percentages. She was, however, not happy with this idea. Next she thought of dividing the total mass by something she could not define. In the end she multiplied the number of moles of $\mathrm{Li}_{3} \mathrm{~N}$ with the mass of Li that would be found in 1 mole of $\mathrm{Li}_{3} \mathrm{~N}$.

## Steps 3 and 4 (calculating the mass of Li)

S2: ... I multiply 1.71 with how much Li will be found in $u$ and arrive at 36 .
I: ... So you arrive at 36 g lithium?
S2: ... Yes, the ratio is 21 to 14. If these are multiplied (by 1.71) and you add up the resulting figures you arrive at 60 g .
I: o.k.
S2: ... Do you understand what I mean?
I: Yes, Ido.
Student 3 solved item 3 using strategy 2. But it was a long way to the solution, and after she worked on item 5 she returned to rework item 3. Student 3 was the only one who immediately recognized that the mass relation $\mathrm{m}(\mathrm{Li}): \mathrm{m}(\mathrm{N})$ was $3: 2$. But because of a mistake in her calculation she first arrived at an incorrect answer.

S3: ... (reads the question and calculates) ... 7 times 3 is $21 \ldots$ and nitrogen $14 \ldots$ we can well see this is a relation of 3 to $2 \ldots 60 \mathrm{~g}$...how much lithium? Hmm ... that's difficult... so the result is 45. Right? Yes, 45 g .
The student gave the impression to be somewhat insecure about her answer.
I: ... Do you think the relation is 3 to 2?
S3: right
I: o.k.
S3: Yes. Can I reason this way?
I: ... Certainly you can.
When she later solved item 5, she used a mathematical strategy, but wanted to work on item 3 again. After some arguing she used strategy 2 to solve item 3.

S 3: The total molecular mass is 36. One nitrogen and one lithium and...the part of the molecular mass that consists of lithium is 21 divided by 35 and is $60 \%$.
I: 21 by 35 is what again?
S3: Well, $60 \%$ which is lithium
I: $60 \%$ is lithium
S3: Mm ...
I: And I multiplied it with the grams.
S3: So you took 0.6 times 60 ?
S3: Yes, exactly.
Student 3 had good reasons to use the logical strategy, but did not dare to rely on herself. So she turned to the mathematical strategy to check if she was right.

When student 4 started solving item 3, he ran into problems because he used the equation $\mathrm{m}(\mathrm{Li}): \mathrm{m}(\mathrm{N})=\mathrm{n}(\mathrm{Li}): \mathrm{n}(\mathrm{N})$ to do the calculation. After the interviewer had asked him for an explanation, he changed his mind and arrived at the correct answer using strategy 3.

S4: ... That was the wrong line of thought. I have to divide it by 7, therefore, we have one fifth so to speak... 21 is three times seven... and 14 is two times seven ... that makes 3 fifths and 2 fifths.
I: And so you get the result that lithium has a mass of 36 .
S3: ... Yes. $1 / 5$ of 60 is 12, and 12 times 3 makes 36.

## Main phase item 5

All students solved this problem correctly using the same strategy as for item 3. Student 2 used strategy 1, students 1 and 3 strategy 2, and student 4 strategy 3 (Table 2).

Student 1 solved item 5 using strategy 2. She showed confidence in doing the calculations.

S1: Hm... Titanium weighs... 48 u. ... oxygen weighs $16 u$. Two oxygen makes 32 ... 48 plus... $32 \ldots$ 80. The percentage is $40 \ldots$ and that multiplied by $60 \ldots$ This is the same procedure as last time.

Student 2 did not set aside her promising method used to solve item 3 and continued with strategy 1. The discussion was taken up again about what kind of figures she was calculating.

S2: Well, the same as above. I sum up... titanium has 48 u and two oxygen ... that makes 32 u....and get 80. And then I divide 60 by 80, and so get how many grams that is... no, I don't know exactly.
I: It is interesting to see the unit you get. You have got grams up here and $u$ down there, so you arrived at $g / u$.
S2: ... Yes, but there is a little mistake there. I am not quite sure about the units. But then I take this and multiply it by the oxygen there. The two oxygen make 32 and... there I get 24 g . And that fits with the units anyway. Although g/u is a bit of a strange unit. But... I got this result.

Student 2 did not realize that she calculated the amount of substance.
Student 3 used the mathematical strategy 2 which she knew to work well. But it was apparently not obvious to her as she could not remember how she solved the earlier problem. She more or less manipulated the figures until they seemed right.

S3: Well, molecular mass. Yes, that is 80, and what is the portion of oxygen then? Did I multiply with 60 or divide by 60 ? That's the problem, what did I do before?
I: (laughs)
S3: But I believe that...(calculates for herself). Yes, that's what I did... so we get that summed up with 80 . What part of it is oxygen? We have 32 divided by 80 makes 0.4 times 60 g , so that is 24 . We can then calculate like this (takes item 3 again). Now we get all this sorted out. Do you understand? 14 plus 21 ... and then we got 21 divided by 35 multiplied with 60 . The result is 36 .

In this situation, she returned to item 3 in order to apply the same strategy to solve item 5 .
Student 4 had not forgotten his logical strategy. He made a quiet, unassuming impression when he reasoned. He set up the relation between the atomic masses of the elements and from there derived the mass relation.

S4: Same procedure as before. The atomic masses and the smallest denominator which is 8 .
I: You have written that titanium weighs $48 u$. And two oxygen are 16 times 2 , which is 32 . And that's why you wrote that titanium is $6 / 10$. Is that right?
S4: Yes, exactly. Now I think I have made a mistake again.
I: You wrote that oxygen is 4/10. Then you took 60 divided by 10 .
S4: That was only to find out what one part is. And then I multiply this by 4.
Student 4 was the only one who used the same strategy throughout all interviews.

## DISCUSSION

Written tests are like snap-shots producing information about (many) individuals with reference to a certain moment. Interviews are like films. They produce in-depth information about (few) individuals by "the unique interaction between the interviewer and the interviewee" (Kvale, 1996, p. 287). "The research interviewer uses him- or herself as a research instrument" (Kvale, 1996, p. 125). In doing so new variables are introduced. As a result different interviewers may come up with different interviews. Hence, great caution needs to be exercised in interpreting information from interview studies. In the present study results are basically corroborated by previous quantitative pencil-paper studies.
"A common critique of interview studies is that...there are too few subjects". As a consequence some qualitative interview studies seem to be based on the assumption: "the more interviews, the more specific". Many of the interview studies "would have profited from having fewer interviews in the study, and from taking more time to prepare the interviews and to analyze them". Qualitative interviews generate "an immense number of observations of single individuals". The focus on single cases makes it possible to "...work out the logic of the relationship between the individual and the situation" (Kvale, 1996, p. 102/3).

The interviews of the present study show in which steps students developed their strategies. All students used the logical strategy 3 to solve item 2. However, most of them used a mathematical strategy when working on a more difficult item. The transcript shows that during the interviews some students attempted two strategies. Student 2, who used strategy 1 to solve item 3, made an attempt to calculate the mass per cent, which is part of strategy 2 . When trying to solve item 3 , student 3 first applied strategy 3 . A mistake in her calculation caused her to abandon it and to continue using strategy 2. Student 4 even started with an incorrect strategy based on $n(L i): n(N)=m(L i): m(N)$, but applied strategy 3 thereafter.

Table 2 reveals that strategies 2 and 3 dominate students' solutions. We have already mentioned that in our study students using strategy 2 calculated the mass fraction or the mass percentage of one of the elements in the compound. In this form strategy 2 comes close to the logical strategy 3 . It seems that students who worked on a more difficult item used their experience solving an easier one. Based on this result we suggest, for classroom teaching, developing strategy 2 (items 3 and 5) from strategy 3 (items 2 and 4) in the following form:

Step 1: Calculate the molar mass of the compound:
one mole of $\mathrm{Li}_{3} \mathrm{~N}$ is composed of three moles of lithium and one mole of nitrogen $\# M\left(L i_{3} N\right)=35 \mathrm{~g} / \mathrm{mol}$.

Step 2: Calculate the mass fraction of one element present in one mole of the compound:
35 g of lithium nitride is composed of 21 g lithium and 14 g of nitrogen. The mass
fraction of lithium is $2 \lg / 35 \mathrm{~g}=3 / 5$ or $60 \% \mathrm{Li}$.
Step 3: Calculate the mass fraction of this element present in the given mass of the compound:

60 g of lithium nitride is composed of $3 / 5 * 60 \mathrm{~g}$ lithium, i.e. 36 g Li .
Of course, an alternative exists depending on the units that are used: reference can be made to the molar masses of the two elements or to the masses of the atoms involved. This alternative to strategy 2 does not refer to a mathematical equation with division signs at both
sides of an equal sign, as it is true for the original form of the strategy. This illustrates the closeness between the alternative of strategy 2 and strategy 3 .

The students who took part in our study struggled hard to solve the problems successfully and to explain to the interviewer their reasoning. However, it was troublesome for them to express themselves. They rarely connected figures with units. The problems that occurred here became clear when this article was prepared for publication, and the students' comments had to be translated into English.

Are students' strategies described here a result of their reflections or was it the interviewer who led them to the answers? Are our results valid? "Validity...pertains to the quality of interviewing itself, which includes a careful questioning as to the meaning of what was said and a continual checking of the information obtained as a validation in situ" (Kvale 1996, p. 237). In our case the interviewer tried to direct students in the problems of interest as little as possible. In order not to be misunderstood and not to misunderstand the interviewees' answers she repeated important questions in the course of the interviews, but varied the formulations. Often the interviewer asked the student to react on how she understood his or her answer. Most questions that were used during the interviews were discussed between the authors of this paper beforehand. Both authors also read the transcripts of the interviews independently. All this increases the validity of the procedure.
"A persistent question posed to interview studies is whether the results are generalizable. In everyday life we generalize...spontaneously. From our experience with one situation...we anticipate new instances, we form expectations of what will happen in other similar situations..." (Kvale, 1996, p. 232).

In qualitative studies analytical generalization plays an important role. "Analytical generalization involves a reasoned judgement about the extent to which the findings from one study can be used as a guide to what might occur in another situation" (Kvale, p. 233). It can be seen from this reasoning that generalizability does not mean that the results can be applied universally. Maybe, the term generalizability is misleading in this context. The question to be answered is: are there reasons to believe that the results obtained in our study reappear in other situations. Let us look for an answer stepwise.

The first part of the present study referring to research question 1, whether Swedish students behave in a similar manner to that described in published papers, was a replication with a sample from another education system. It showed that the four Swedish students solved the stoichiometry problems in the same way as described in the literature. We, therefore, assume that this part of our results may be applicable beyond our sample, i.e. that students from other upper secondary schools in Sweden may solve the items in a similar way.

Research question 2, how students change strategies when moving from an easy-tocalculate problem to a more difficult question, however, was not previously investigated in another country. Therefore a generalization beyond the population of our four students cannot be made. However, after reading how our students employed logical reasoning in their solutions, researchers and teachers might expect similar logical reasoning in their own students and conduct a study similar to ours.

## IMPLICATIONS FOR RESEARCH AND TEACHING

More research is needed to improve the validity of the procedure and the generalizability with respect to other groups and other problems. In the items used in this study a chemical formula is given. There are other easy-to-calculate items in which a chemical formula has to be developed (Schmidt, 1994, 1997). It would be interesting to know
which strategies students use when moving for example from item 2 and /or 3 to items which require the development of a chemical formula from given masses. It also remains to be studied how students proceed moving from item 2 and/or 3 to item 1 (and similar problems). Another aspect to study: how do students move from an erroneous strategy to strategy 3? In our study student 4 started doing the calculation this way but gave it up immediately. Finally, studies as the one described should not only be done in Sweden, but also in other countries.

Research should not aim at developing "the" teaching strategy. Due to the individuality no one teaching strategy would fit every student. However, there is good reason to believe that in introductory courses on stoichiometry many students may profit from learning about strategy 3 and strategy 2 (as desribed in the section above).

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