

New Coupling Scheme in Heavy Nuclei

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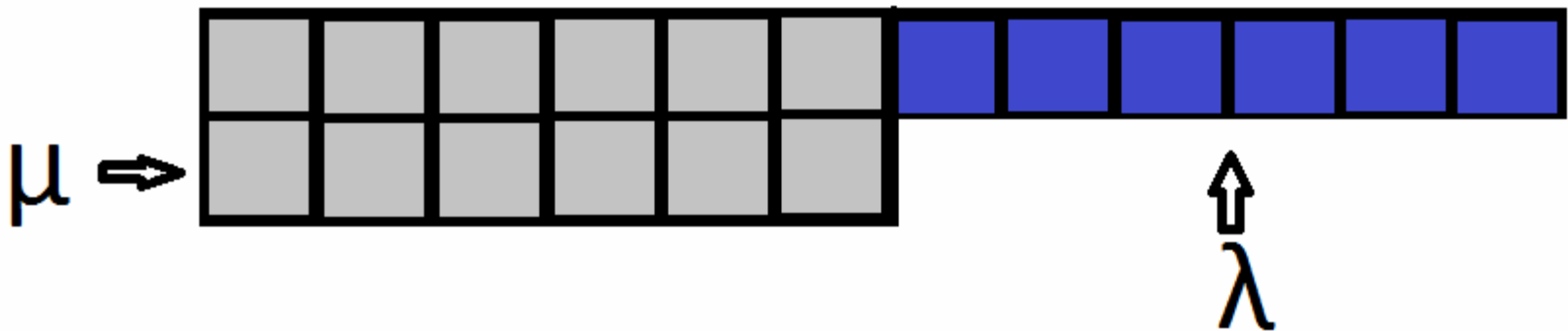
NEW COUPLING SCHEME

- SU(3) symmetry of the harmonic oscillator is destroyed in heavy nuclei due to spin-orbit interaction
- However the SU(3) symmetry is present at heavy nuclei
- The “partnership” between $0[110]$ orbitals leads to a scheme conserving the SU(3) symmetry

THE UNITARY GROUP AND ITS IRREDUCIBLE REPRESENTATIONS

- The unitary group $U(n)$ is the group of n dimensional unitary matrices
- The “pf” shell of a nucleus has $U(10)$ symmetry
- The “sdg” shell of a nucleus has $U(15)$ symmetry
- Every shell of a nucleus can be described using a unitary group of an appropriate dimension

- ❑ In order to describe a specific nucleus we use irreducible representations of $U(n)$ groups
- ❑ Irreducible representations (=irreps) conserve good quantum numbers
- ❑ The irreps of $SU(3)$ (which is a subgroup of $U(n)$) used are denoted by (λ, μ)
- ❑ They can be described graphically by a Young diagram



- λ and μ have to do with the deformation of the nucleus
- $\lambda \rightarrow$ magnitude of deformation
- $\mu \rightarrow$ axial or triaxial shape
- We need only the irreducible representations (λ, μ) which maximize the quantity

$$\frac{1}{9}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$$

- For example, ^{168}Er has 18 valence protons and 18 valence neutrons
- The most leading $\text{SU}(3)$ irrep of $\text{U}(10)$ corresponding to the protons is $(10,4)$
- The most leading $\text{SU}(3)$ irrep of $\text{U}(15)$ corresponding to the neutrons is $(20,4)$
- So the nucleus ^{168}Er can be described by the $(30,8)$ irrep

OPERATORS OF THE NEW HAMILTONIAN

➤ The second order Casimir operator of $SO(3)$ \hat{L}^2 with eigenvalues $I_2(L) = L(L + 1)$

➤ The second order Casimir operator of $SU(3)$ with eigenvalues

$$C_2(\lambda, \mu) = \frac{1}{9}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$$

➤ The third Casimir operator of $SU(3)$ with eigenvalues

$$C_3(\lambda, \mu) = \frac{1}{162}(\lambda - \mu)(2\lambda + \mu + 3)(\lambda + 2\mu + 3)$$

- The third order scalar shift operator of $O(3)$ called Ω in nuclear physics literature.
- The fourth order scalar shift operator of $O(3)$ called Λ in nuclear physics literature.
- Ω and Λ will break the degeneracy between the bands belonging to the same irrep.
- There is no analytical expression for the eigenvalues of Ω and Λ . Numerical calculations must be performed.

- Including only one body and two body terms leads to a Hamiltonian with eigenvalues

$$E = \alpha L(L+1) + C_2(\lambda, \mu)$$

In this scheme bands belonging to the same irrep are degenerate

- Including terms up to third order leads to a Hamiltonian with eigenvalues

$$E = \alpha L(L+1) + \beta C_2(\lambda, \mu) + \gamma \Omega + \delta C_3(\lambda, \mu)$$

Ω breaks the degeneracy. $C_3(\lambda, \mu)$ term can be omitted.

This is the lowest energy Hamiltonian of the new scheme.

- Including up to fourth order terms leads to a Hamiltonian with eigenvalues

$$E = \alpha L(L+1) + \beta C_2(\lambda, \mu) + \gamma \Omega + \delta C_3(\lambda, \mu) + \xi \Lambda + \\ + \nu L^2(L+1)^2 + \tau L(L+1)C_2(\lambda, \mu) + \rho [C_2(\lambda, \mu)]^2$$

The terms $C_3(\lambda, \mu)$, $L(L+1)C_2(\lambda, \mu)$, $[C_2(\lambda, \mu)]^2$ can be omitted

Ongoing numerical calculations for Ω and Λ

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