Speed of Sound Effects on the Upper Bound of Non-Rotating Neutron Star Mass

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Outline

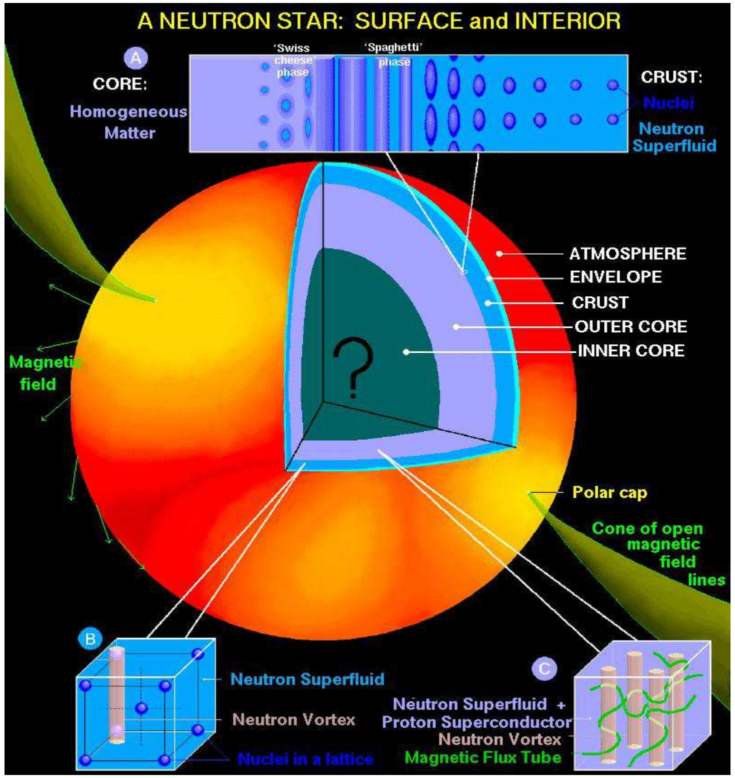
A. Introduction on neutron star structure and nuclear equation of state

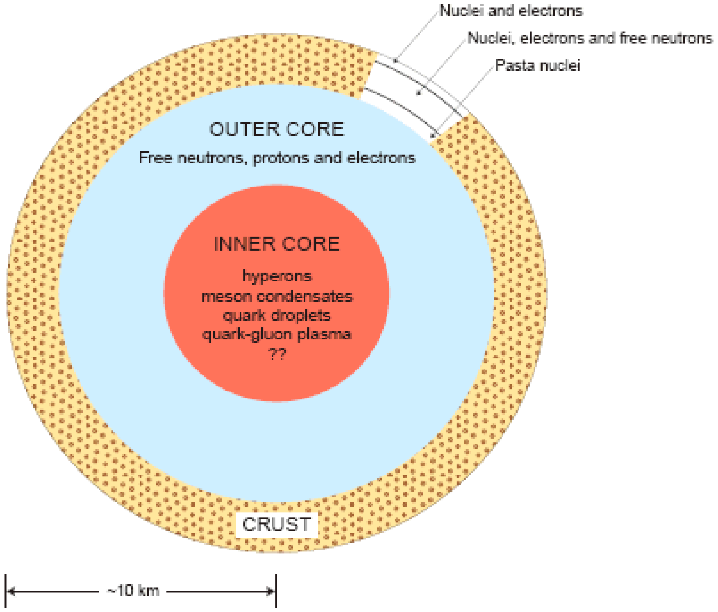
B. The speed of sound in nuclear matter and relative constraints. Implication on the maximum neutron star mass (MNSM)

C. Relativistic kinetic theory constraints on MNSM

E. Presentation of the results

D. Main conclusions and perspectives





Properties of Neutron Stars

Radius: R~10-15 km
Mass: M~1.4–2.5 Msolar
Mean density: ρ(r)~4x10^14 g/cm3
frequency: few Hz– 700 Hz
Magnetic field: B~100–1000 G

The basic assumptions

We consider the assumptions[1]

1. the matter of the neutron star is a perfect fluid described by a one-parameter equation of state between the pressure P and the density ρ
2. the density ρ is non negative (due to attractive character of gravitational forces)
3. the matter is microscopically stable, which is ensured by the conditions $P \geq 0$ and $dP/d\rho \geq 0$
4. below a critical density ρ_c the equation of state is well known

From the above assumptions and the TOV equations it follows that the density and pressure decreases outward in the star. In addition the radius R_c at which the pressure is $P_c = P(\rho_c)$ divides the neutron star into two regions. The core where $r \leq R_c$ and $\rho \geq \rho_c$ and the envelope where $r \geq R_c$ and $\rho \leq \rho_c$.

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[1] C.E. Rhoades Jr. and R. Ruffini, Phys. Rev. Lett. **32**, 324 (1974)

The upper limits of the speed of sound

The speed of sound is defined as

$$\frac{v_s}{c} = \left(\frac{\partial P}{\partial \mathcal{E}}\right)_S^{1/2}$$

We consider the following three limits for the speed of sound

1. $\frac{v_s}{c} \leq 1$: causality limit from special relativity[2]

2. $\frac{v_s}{c} \leq \frac{1}{\sqrt{3}}$: from QCD and other theories[3]

3. $\frac{v_s}{c} \leq \left(\frac{\mathcal{E} - P/3}{P + \mathcal{E}}\right)^{1/2}$: from relativistic kinetic theory[4]

[2] J.B. Hartle, Phys. Rep. **46**, 201 (1978)
[3] P. Bedaque and A.W. Steiner, Phys.Rev.Lett. **114**, 031103 (2015)
[4] T.S. Olson, Phys. Rev. C **63**, 015802 (2002)

The knowledge of maximum neutron star mass in important since:

1. **Helps to identify a compact object as a black hole.**

2. **The accurate calculation of MNSM strongly depends on the knowledge of the nuclear equation of state up to very high densities.**

3. **Related with the appearance of hyperons and other degrees of freedom.**

4. **Helps to understand some of the more extreme NS related processes like core-collapse supernovae, magnetar flares, and NS mergers.**

Nuclear equation of state models

The momentum dependent interaction model (Skyrme type)

$$
\begin{aligned}
E_b(n,I) = {} & \frac{3}{10}E_F^0 u^{2/3}\left[(1+I)^{5/3}+(1-I)^{5/3}\right]+\frac{1}{3}A\left[\frac{3}{2}-(\frac{1}{2}+x_0)I^2\right]u \\
+ & \frac{\frac{2}{3}B\left[\frac{3}{2}-(\frac{1}{2}+x_3)I^2\right]u^{\sigma}}{1+\frac{2}{3}B'\left[\frac{3}{2}-(\frac{1}{2}+x_3)I^2\right]u^{\sigma-1}} \\
+ & \frac{3}{2}\sum_{i=1,2}\left[C_i+\frac{C_i-8Z_i}{5}I\right]\left(\frac{\Lambda_i}{k_F^0}\right)^3\left(\frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}-\tan^{-1}\frac{((1+I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right) \\
+ & \frac{3}{2}\sum_{i=1,2}\left[C_i-\frac{C_i-8Z_i}{5}I\right]\left(\frac{\Lambda_i}{k_F^0}\right)^3\left(\frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}-\tan^{-1}\frac{((1-I)u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right).
\end{aligned}
$$

The non linear derivative model (NLD)

(Non-linear derivative interactions in relativistic hadrodynamics:

T. Gaitanos, M. Kaskulov and U. Mosel, NPA 828, 9-28 (2009))

Microscopic model of neutron mater based on nuclear interaction derived from chiral effective field theory (CEFT): Hebeler et al., PRL, 105 161102 (2010).

$$
\frac{E(u,x)}{T_0}=\frac{3}{5}\left[x^{5/3}+(1-x)^{5/3}\right](2u)^{2/3}-\left[(2\alpha-4\alpha_L)x(1-x)+\alpha_L\right]u+\left[(2\eta-4\eta_L)x(1-x)+\eta_L\right]u^{\gamma}
$$

The TOV Equations

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P(r)r^3}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2 r}\right)^{-1}$$

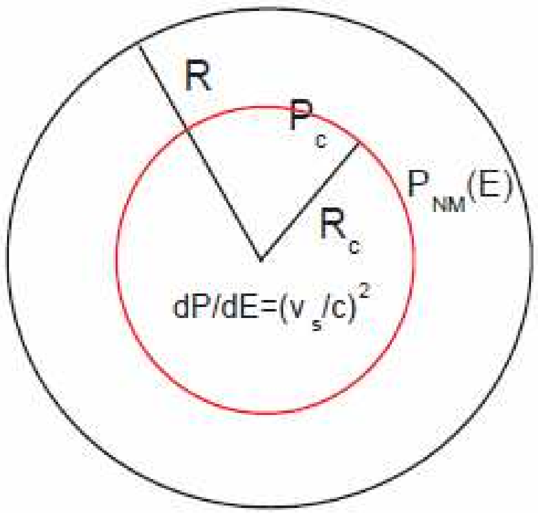
$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\mathcal{E} = n\left(E + mc^2\right) = \rho c^2 \qquad P = n\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}n} - \mathcal{E}$$

To solve the TOV equations for $P(r)$ and $M(r)$ one can integrate outwards from the origin ($r = 0$) to the point $r = R$ where the pressure becomes zero. This point defines R as the coordinate radius of the star. To do this, one needs an initial value of the pressure at $r = 0$, called $Pc = P(r = 0)$. The radius R and the total mass of the star, $M \equiv M(R)$, depend on the value of Pc. To be able to perform the integration, one also needs to know the energy density $\mathcal{E}(r)$ (or the density mass $\rho(r)$) in terms of the pressure P(r). This relationship is the equation of state for neutron star matter.

The implication of the speed of sound

$$P(\mathcal{E}) = \begin{cases} P_{NM}(\mathcal{E}), & \mathcal{E} \leq \mathcal{E}_c \\ \left(\frac{vs}{c}\right)^2 (\mathcal{E} - \mathcal{E}_c) + P_{NM}(\mathcal{E}_c), & \mathcal{E} \geq \mathcal{E}_c. \end{cases}$$



$$v_s = c\sqrt{\frac{\partial P}{\partial \mathcal{E}}}$$

The relativist kinetic theory constraints

$$\mathcal{E} \geq 0, \qquad P \geq 0, \qquad (P+\mathcal{E})\left(\frac{v_s}{c}\right)^2 \geq 0, \qquad \left(\frac{v_s}{c}\right)^2 \leq \frac{\mathcal{E}-P/3}{P+\mathcal{E}}, \qquad P \leq 3\mathcal{E}$$

The maximally incompressible EOS is taken from the equality

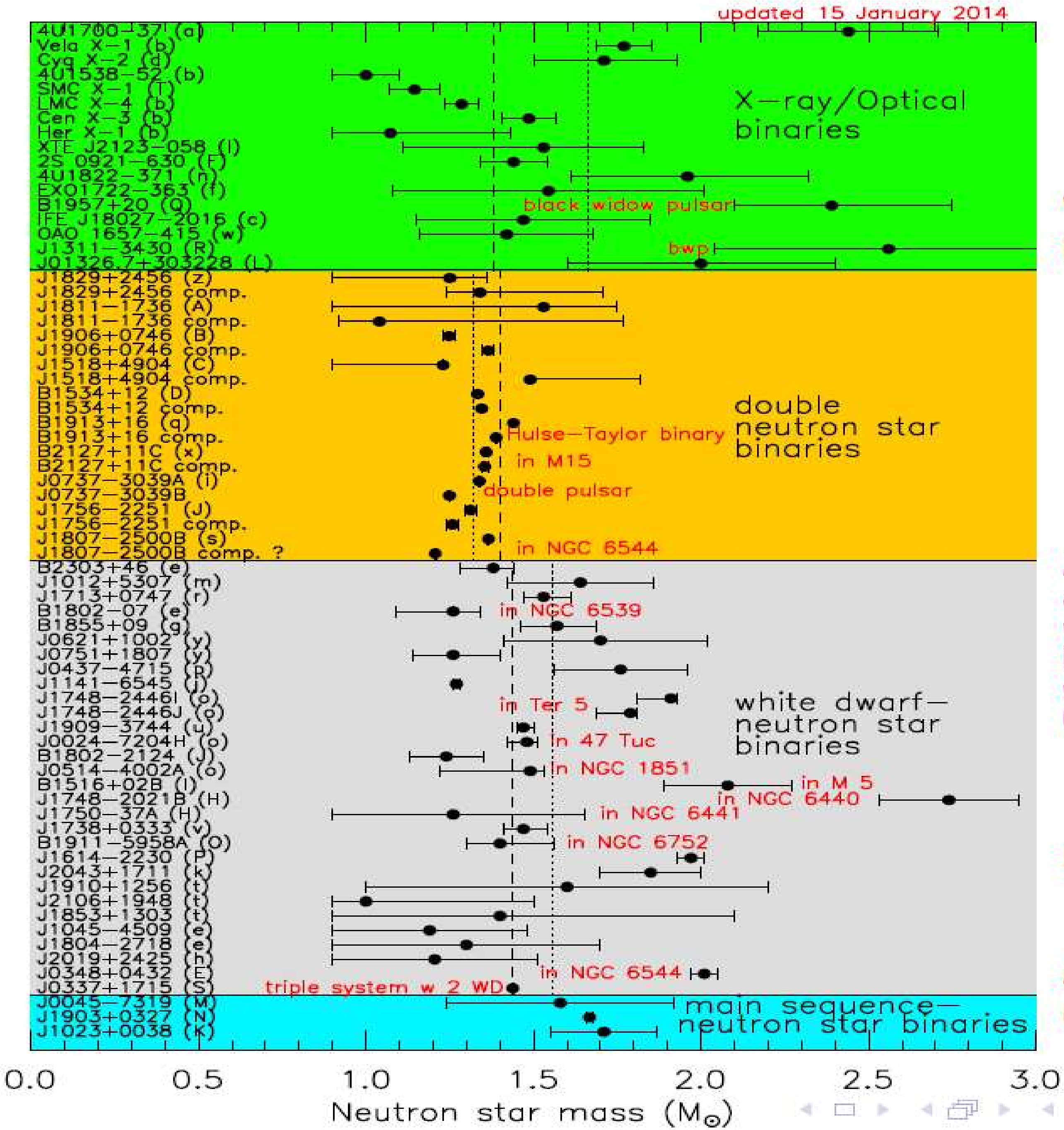
$$\left(\frac{v_s}{c}\right)^2 = \frac{\mathcal{E}-P/3}{P+\mathcal{E}} \qquad\qquad n^2\frac{d^2\mathcal{E}}{dn^2} + \frac{n}{3}\frac{d\mathcal{E}}{dn} - \frac{4}{3}\mathcal{E} = 0$$

$$\mathcal{E}(n) = \mathcal{C}_1 n^{a_1} + \mathcal{C}_2 n^{a_2} \qquad\qquad P(n) = \mathcal{C}_1 n^{a_1}(a_1-1) + \mathcal{C}_2 n^{a_2}(a_2-1),$$

$$\mathcal{C}_1 = \left(\frac{(2+\sqrt{13})\mathcal{E}(n) + 3P(n)}{2\sqrt{13}}\right) n^{-\frac{1}{3}(1+\sqrt{13})}$$

$$\mathcal{C}_2 = -\left(\frac{(2-\sqrt{13})\mathcal{E}(n) + 3P(n)}{2\sqrt{13}}\right) n^{-\frac{1}{3}(1-\sqrt{13})}$$

The values of the constants $\mathcal{C}_1$ and $\mathcal{C}_2$ are determined my the help of the matching density n_c.



vanKerkwijk 2010

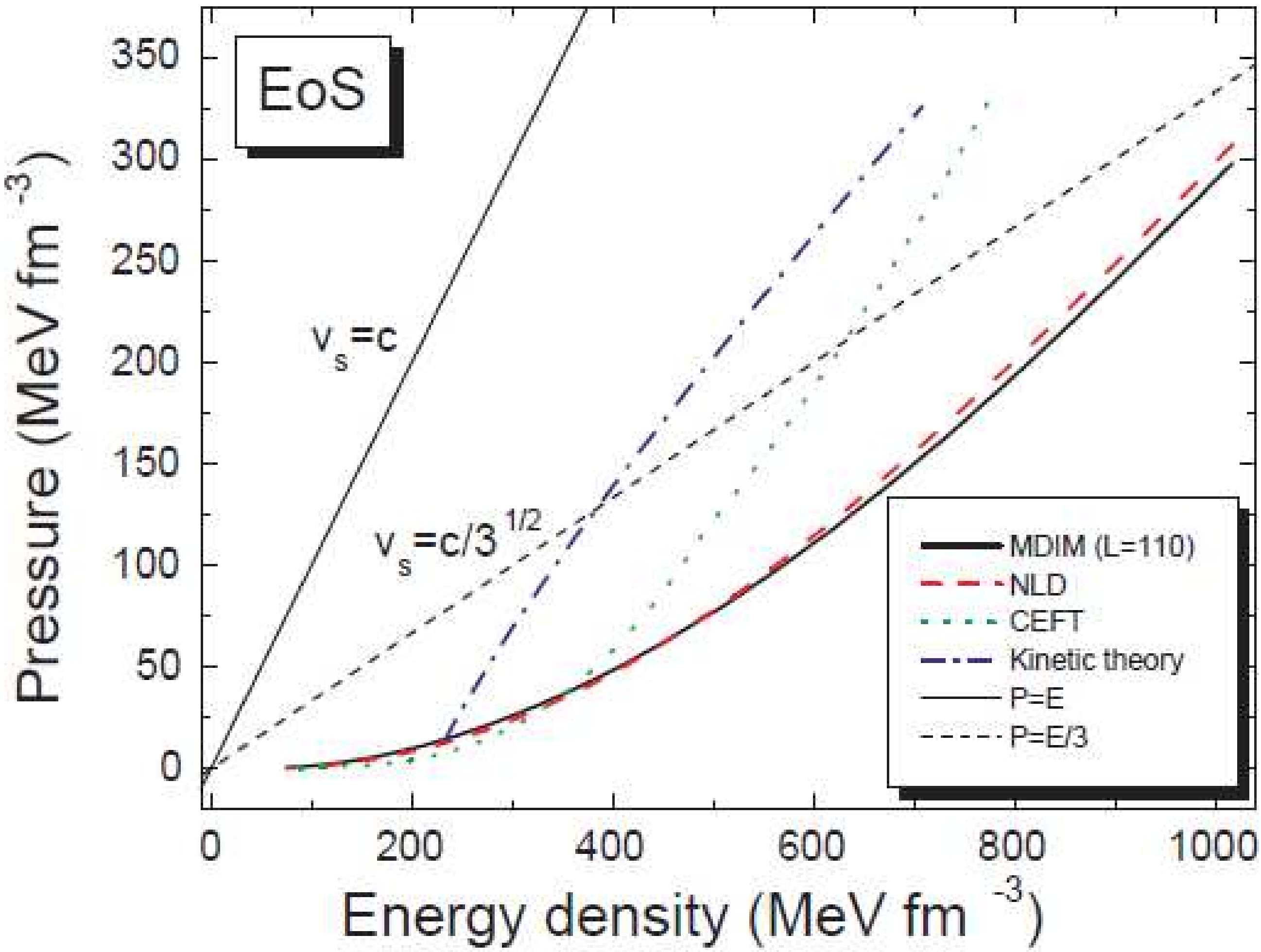
Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

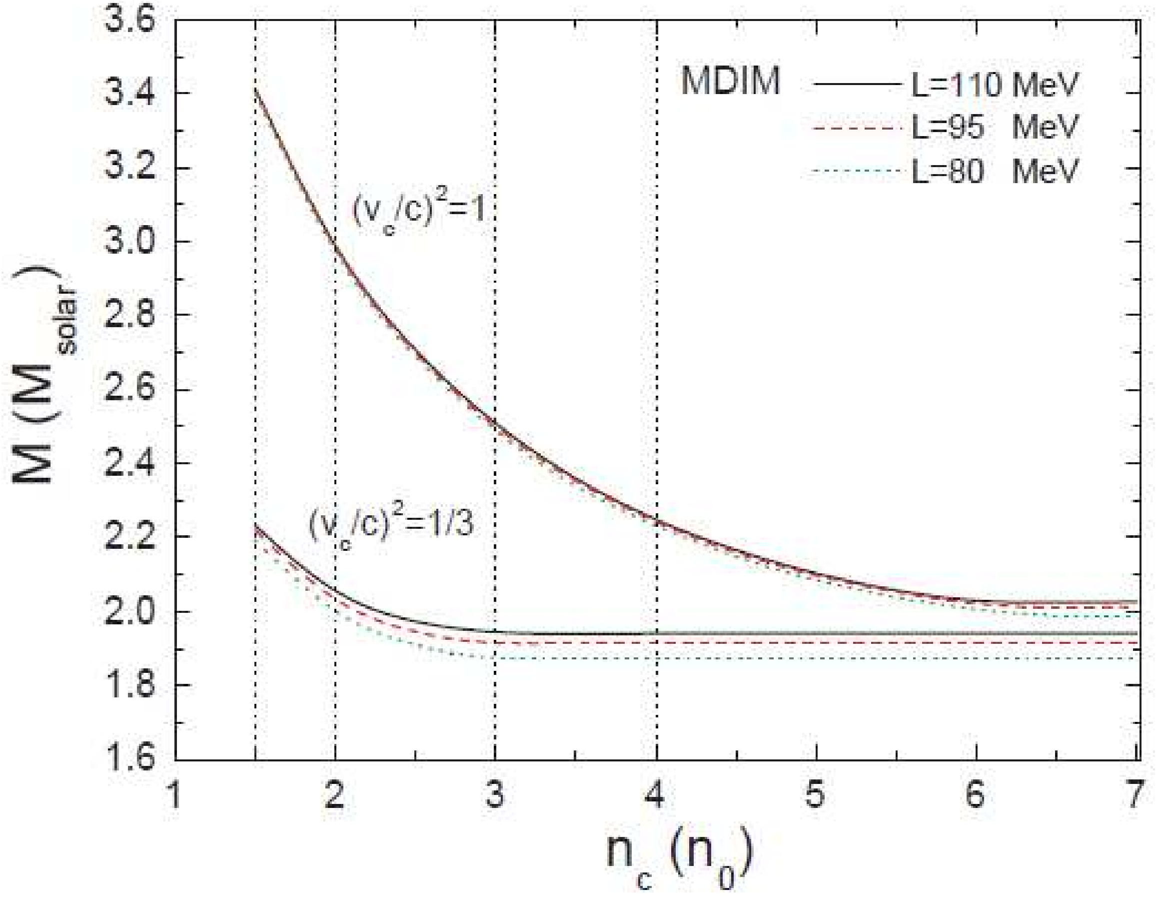
Demorest et al. 2010

Antoniadis et al. 2013

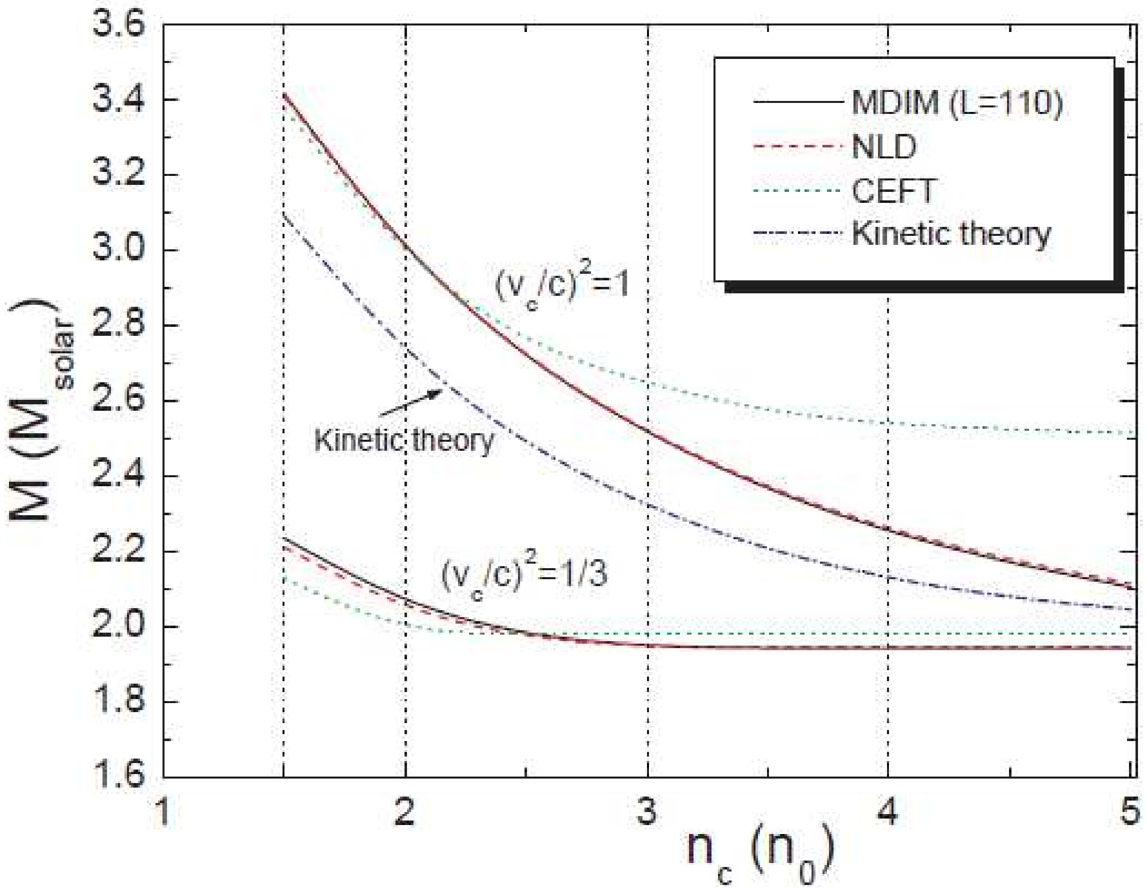
Champion et al. 2008



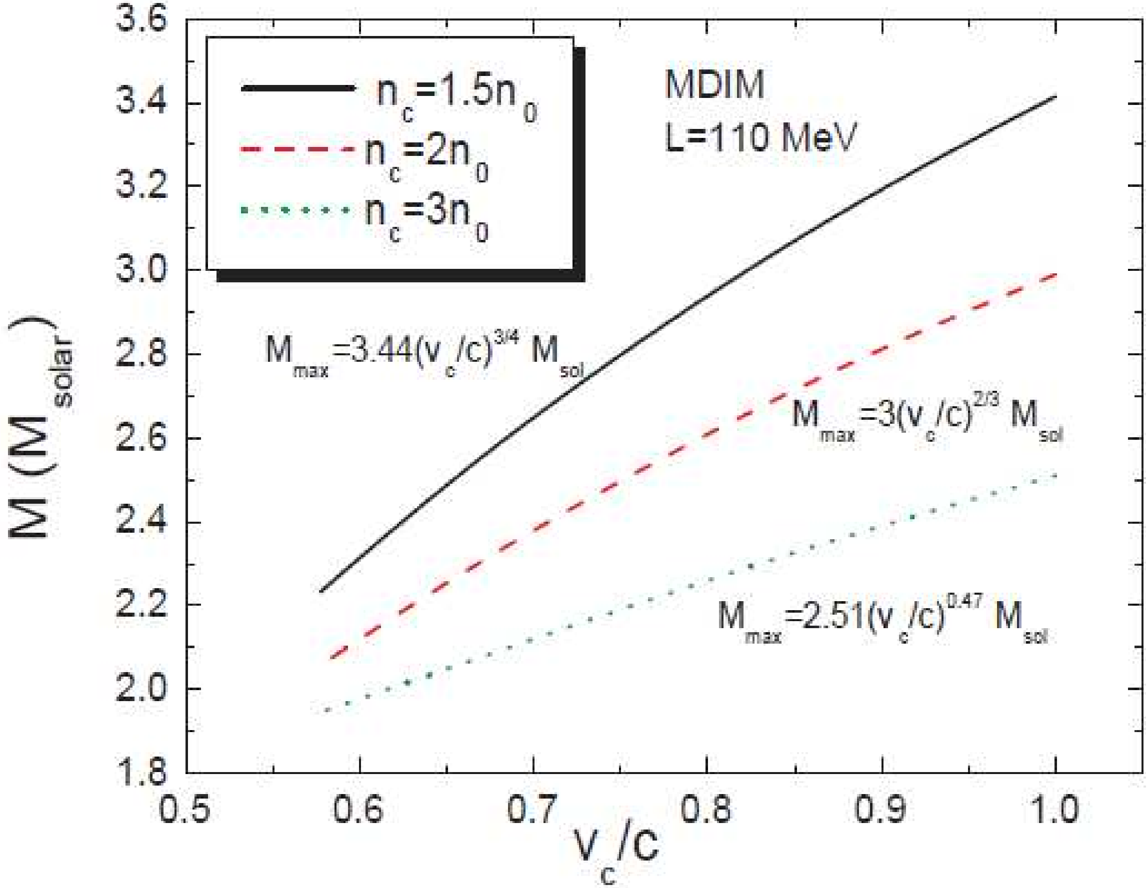
EoS
Pressure (MeV fm -3)
Energy density (MeV fm -3)
v_s=c
v_s=c/3 1/2
MDIM (L=110)
NLD
CEFT
Kinetic theory
P=E
P=E/3
0
50
100
150
200
250
300
350
0
200
400
600
800
1000



MDIM
L=110 MeV
L=95 MeV
L=80 MeV
$(v_c/c)^2=1$
$(v_c/c)^2=1/3$
M (M_{solar})
n_c (n_0)
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1
2
3
4
5
6
7



MDIM (L=110)
NLD
CEFT
Kinetic theory
$(v_c/c)^2=1$
$(v_c/c)^2=1/3$
Kinetic theory
M (M_{solar})
n_c (n_0)



n_c=1.5n_0
n_c=2n_0
n_c=3n_0
MDIM
L=110 MeV
$M_{max}=3.44(v_c/c)^{3/4}\ M_{sol}$
$M_{max}=3(v_c/c)^{2/3}\ M_{sol}$
$M_{max}=2.51(v_c/c)^{0.47}\ M_{sol}$
M (M_{solar})
v_c/c
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
0.5
0.6
0.7
0.8
0.9
1.0

Conclusions-Outlook

- The knowledge of MNSM is important in order to identify the low mass black holes especially in binary systems
- The upper bound of the speed of sound is still an open problem in strongly interactive systems.
- It is conjectured that the limit vs=c/3^(1/2) represents an absolute upper bound for a broad class of theories. One has to examine the effects of this bound on neutron star properties and mainly on the MNSM
- The stiffness of the equation of state strongly constrained by the upper limit of the speed of sound. The upper limit vs=c is compatible with maximum mass up to 3 Msolar
- However the limit vs=c/3^(1/2) is in contradiction with the recent measurements of neutron star with mass close to 2 Msolar.
- The relativistic kinetic theory predict Mmax close to the value 2.7 Msolar.

General Perspectives

1) The maximum and the minimum limit of neutron star mass related with the nuclear equation of state at supranuclear and subnuclear densities respectively

2) Precise measurements of masses and radii for several individual neutron stars would pin down the equation of state without recourse to models

3) The phase diagram of dense matter at low temperature. The hadrons-quark phase transition at high densities.

4) The link between the microphysics of transport, heat flow, superfluidity, viscosity, vortices tubes and the macro-modes in neutron star phenomenology

5) Measuring the neutron star equation of state with gravitational wave observations