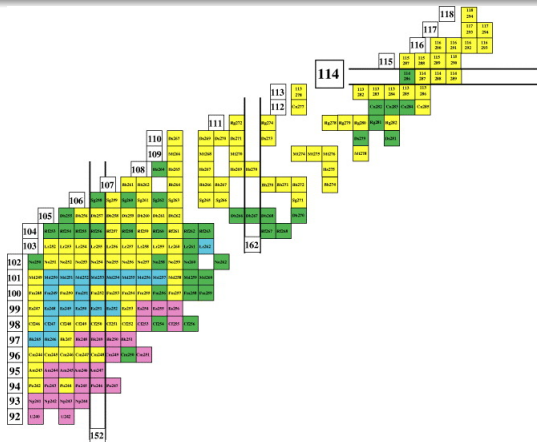


- Elements with **atomic number greater than Uranium ($Z=92$)** exist due to shell effects. Balance between nuclear force and coulomb field.
- Prediction of an **island of stability** around $Z=114$ and $N=184$ in the 1960's.
- Theoretical studies were based on the traditional *macroscopic-microscopic approach* for many years. Since the late 1990s SCMF based on the *Gogny interaction*, the *Skyrme energy functional*, and the *relativistic meson exchange effective Lagrangians* have systematically been applied to the structure of SHN.
- Models predict **rapid shape transitions**.



- Compound nucleus reactions between ^{48}Ca beam and actinide targets.
- New elements with $Z = 113 - 118$ have been synthesized, and new isotopes of Ds ($Z=110$) & Cn ($Z=112$) have been identified.
- Decay energies & T_α provide evidence of a significant increase of stability with increasing neutron number in this region of SHN.

Energy density functional framework

Relativistic Hartree-Bogoliugov framework

Unified treatment of the **nuclear MF** (particle-hole (ph)) and pairing (particle-particle (pp)) correlations

$$\mathcal{E} = \mathcal{E}_{RMF}[j_{\mu}, \rho_s] + \mathcal{E}_{pp}(\kappa, \kappa^*)$$

DD-PC1

DD-PC1 functional fitted to the experimental masses of 64 axially deformed nuclei in the regions $A \approx 150 - 180$ and $A \approx 230 - 250$.

Energy density functional framework

Relativistic Hartree-Bogoliugov framework

Unified treatment of the nuclear MF (particle-hole (ph)) and **pairing** (particle-particle (pp)) correlations

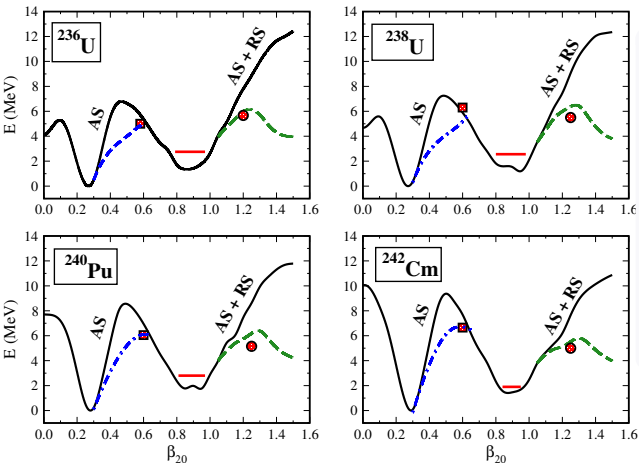
$$\mathcal{E} = \mathcal{E}_{RMF}[j_{\mu}, \rho_s] + \mathcal{E}_{pp}(\kappa, \kappa^*)$$

Pairing interaction: finite range separable pairing

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}'_1, \mathbf{r}'_2) = G\delta(\mathbf{R} - \mathbf{R}')P(\mathbf{r})P(\mathbf{r}')\frac{1}{2}(1 - P^{\sigma})$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad P(\mathbf{r}) = \frac{1}{4\pi a^2} e^{-\frac{r^2}{4a^2}}$$

Test-Fission barriers of actinides



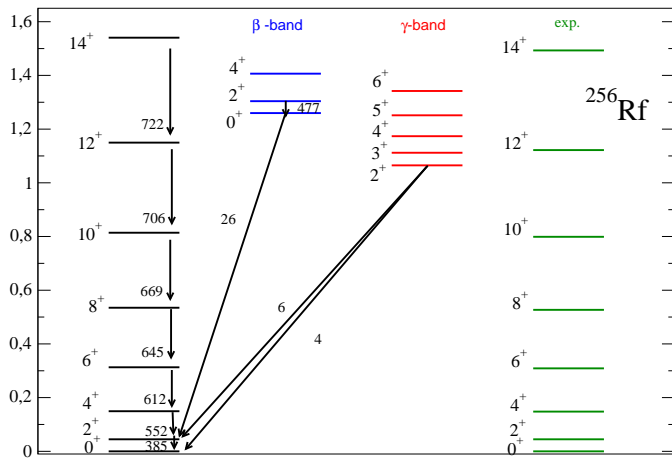
Potential energy surfaces of actinides:

Black: Axial symmetric calculations.

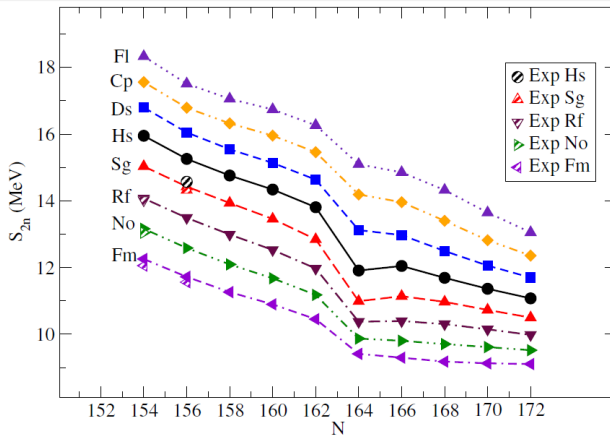
Blue: Triaxial calculations.
Decrease of the first barrier.

Green: Axial symmetric calculations including reflection asymmetry (*octupole*).
Decrease of the second barrier.

Structure of transactinides

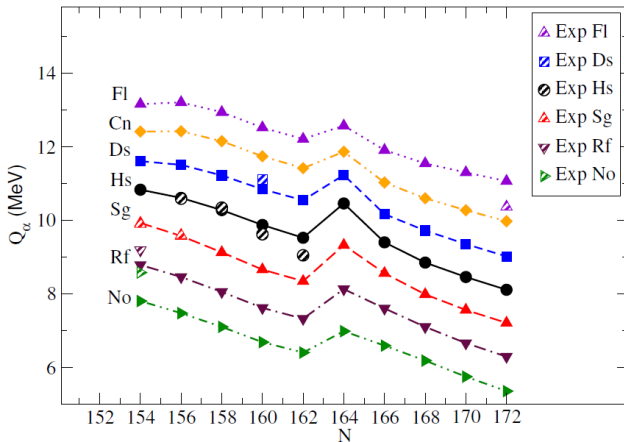


Two-neutron separation energies



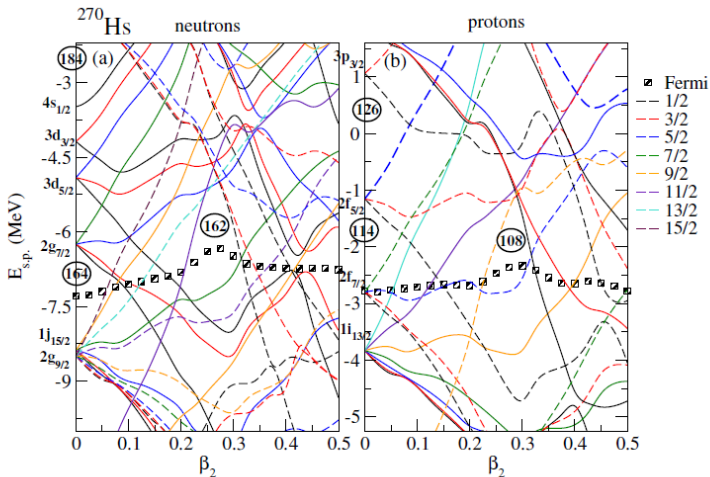
A **sharp decrease** at **$N=162$** discloses a change in the structure of s.p. levels of these isotopes.

Quantitative agreement between model predictions and available data.

Q_α -values

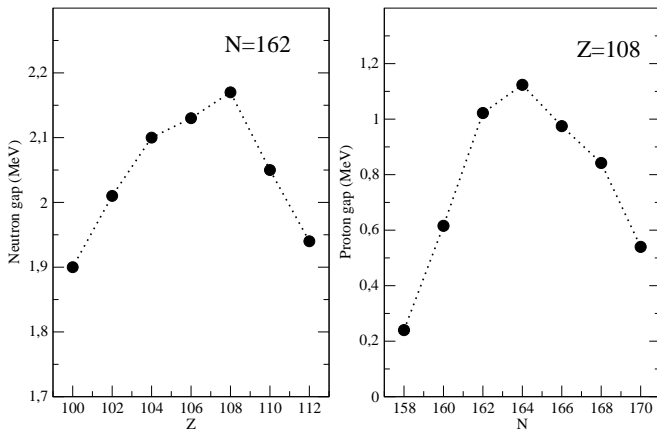
Manifestation of **local minima at N=162**.

Agreement between the calculated Q_α -values and data at the same level as with mic-mac or self-consistent calculations with the Gogny force or the Skyrme functionals.



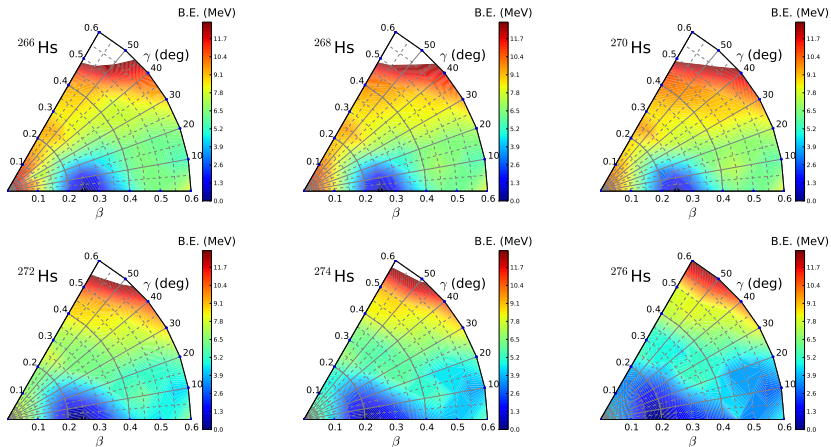
Neutron s.p.e. (left panel) a *prolate deformed gap* ≈ 2.17 MeV at $N=162$ determined by the high- j orbitals $1j_{15/2}$ and $2g_{9/2}$.

Proton s.p.e. (right panel) show a smaller gap at $Z=108$, ≈ 1 MeV.



Neutron gap (left panel) reaches its maximum at $Z = 108$

Proton gap (right panel) peaks at $N=162-164$



Lighter systems ($158 < N < 164$) *deep, prolate MF minima* ($\beta_{20} \approx 0.25$). The inner fission barriers reach values of ≈ 8 MeV.

Intermediate nuclei ($N \approx 164$) exhibit signatures of *triaxiality*.

Heaviest systems (very neutron rich, $N > 166$) display very *soft axially deformed shapes* with minima that extend from the spherical configuration up to $|\beta_{20}| \approx 0.5$.

Nuclear Many-Body Correlations



short-range

(hard repulsive core of the NN-interaction)

long-range

nuclear resonance modes
(giant resonances)

collective correlations

large-amplitude soft modes:
(center of mass motion, rotation,
low-energy quadrupole vibrations)

...vary smoothly with nucleon number!
Implicitly included in an effective EDF.

...sensitive to shell-effects and strong variations with nucleon number!

Cannot be included in a simple Kohn-Sham EDF framework!

Collective Bohr Hamiltonian

... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom.

The entire dynamics of the collective Hamiltonian is governed by the seven functions of the intrinsic deformations β and γ : the three moments of inertia I_k , the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the collective potential .

$$\mathcal{H}_{coll} = \mathcal{T}_{rot} + \mathcal{T}_{vib} + \mathcal{V}_{coll}$$

with the **rotational** and vibrational kinetic energy and the collective potential energy terms.

Rotational energy

$$\mathcal{T}_{rot} = \frac{1}{2} \sum_{k=1}^3 \frac{\hat{J}_k^2}{\mathcal{I}_k}$$

The moments of inertia are calculated by using the **Inglis-Belyaev formula**.

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$$\mathcal{H}_{coll} = \mathcal{T}_{rot} + \mathcal{T}_{vib} + \mathcal{V}_{coll}$$

with the rotational and **vibrational** kinetic energy and the collective potential energy terms.

Vibrational energy

$$\begin{aligned} \mathcal{T}_{vib} = & -\frac{\hbar^2}{2\beta^4\sqrt{wr}} \left[\partial_\beta \sqrt{\frac{r}{w}} \beta^4 B_{\gamma\gamma} \partial_\beta - \partial_\beta \sqrt{\frac{r}{w}} \beta^3 B_{\beta\gamma} \partial_\gamma \right] \\ & -\frac{\hbar^2}{\sin 3\gamma\sqrt{wr}} \left[-\frac{1}{\beta^2} \partial_\gamma \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \partial_\beta + \frac{1}{\beta} \partial_\gamma \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\beta} \partial_\gamma \right] \end{aligned}$$

The mass parameters are calculated in the **cranking approximation**.

Collective Bohr Hamiltonian

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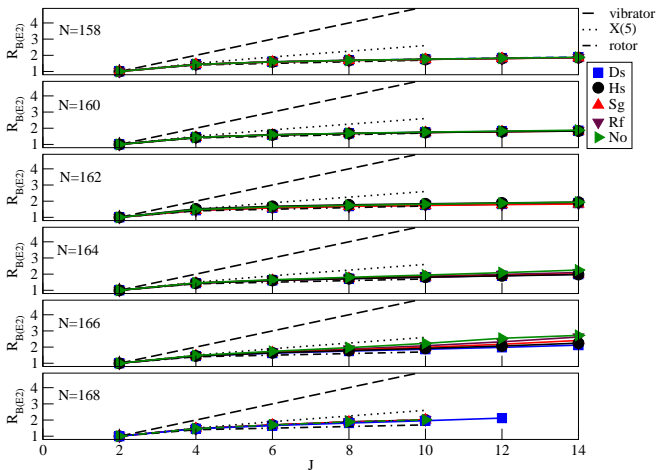
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with the rotational and vibrational kinetic energy and the **collective potential energy** terms.

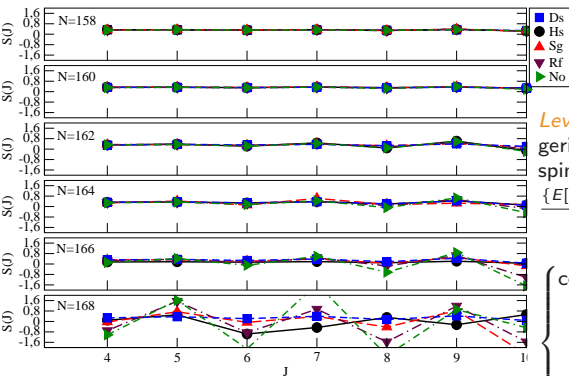
Collective potential

$$\mathcal{V}_{coll}(\beta, \gamma) = E_{tot}(\beta, \gamma) - \Delta V_{vib}(\beta, \gamma) - \Delta V_{rot}(\beta, \gamma)$$

Corresponds to the mean-field potential energy surface with the zero point energy subtracted.



For $N < 164$ all five isotopic chains display *rotor characteristics* close to the SU(3) symmetry limit. For $N > 164$ the value of R_{BE2} *increases towards the vibrator* limit, consistent with the shape evolution of more γ -soft configurations and low values of inner fission barriers.



Level of K -mixing \Rightarrow energy staggering between the odd- and even-spin states in the (quasi) γ -bands:

$$\frac{\{E[J_{\gamma}^+] - E[(J-1)_{\gamma}^+]\} - \{E[(J-1)_{\gamma}^+] - E[(J-2)_{\gamma}^+]\}}{E[2_1^+]} =$$

{ constant } axially symmetric rotor
 { - even-spin } deformed γ -soft potential
 { + odd-spin }
 { + even-spin } triaxial γ -soft potential
 { - odd-spin }

For $N < 164$: constant \Rightarrow $SU(3)$ rotor.

For $N > 164$: more pronounced K -mixing increases with N . Oscillates between positive values for odd-spin and negative for the even-spin states \Rightarrow γ -soft potential.

Summary:

The structure of transactinide nuclei has been analyzed using a *self-consistent formalism* based on NEDF.

Calculations of even-even nuclei with $Z > 90$, using *axial* and *triaxial* implementations of the RHB model and with a *collective Bohr Hamiltonian*, beyond the MF approximation.

Conclusions:

- A discontinuity at $N=162$ is predicted in the calculations of **two-neutron separation energies** and **Q_α values** of transactinides with $Z = 100 - 114$, associated with the neutron gap at $N = 162$ in the s.p.e of ^{270}Hs .
- **Triaxial potential energy surfaces** of nuclei with $N = 158 - 164$ display deep prolate minima.
- **Beyond MF:** The **relative $B(E2)$ values** for $K = 0^+$ **inraband transitions**, and the energy **staggering in the $K = 2^+$ band**, show that the lighter nuclei are rotors close to the $SU(3)$ symmetry limit, whereas the potentials of heavier isotopes become γ -soft.

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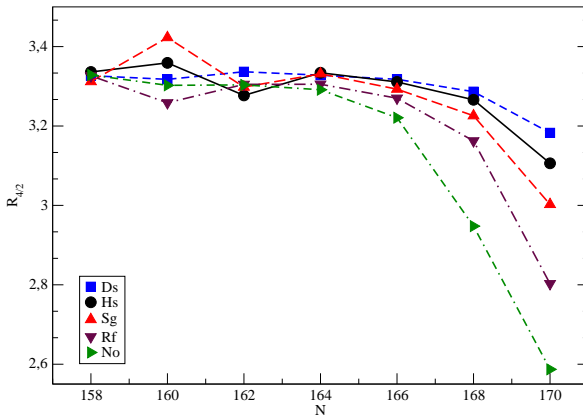
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THANK YOU FOR YOUR ATTENTION



IN COLLABORATION WITH:

D. VRETENAR, T. NIKŠIĆ, PMF, University of Zagreb
G.A. LALAZISSIS, Aristotle University of Thessaloniki



For $N < 164$, $R_{4/2} \approx 3.33$ *rotor behavior* close to the SU(3) symmetry limit.

For $N > 164$ the value of $R_{4/2}$ *decreases towards the vibrator* ≈ 2.0 limit, consistent with the shape evolution of more γ -soft configurations and low values of inner fission barriers.