Exclusive muon capture rates

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Overview

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Weak Interaction Processes

Muon capture is a charged-current weak interaction process in nuclei like:

- $\beta^{\pm}$-decay :

$$(A,Z) \rightarrow (A,Z+1) + e^{-} + \nu_e$$
$$(A,Z) \rightarrow (A,Z-1) + e^{+} + \bar{\nu_e}$$

- lepton capture ($\ell = e^{-}, \mu^{-}, \tau^{-}$):

$$(A,Z) + e^{-} \rightarrow (A,Z-1)^{*} + \nu_e$$

Orbital muon capture in nuclei

$$\mu_b^{-} + (A,Z) \rightarrow (A,Z-1)^{*} + \nu_\mu$$

is an important μ^{-}-capture channel useful:

- in nuclear structure calculations (transition ME, nuclear spectra)
- in testing nuclear models employed for nuclear applications and astrophysics.

Formalism for μ^--capture rates

The starting point is the **weak interaction Hamiltonian**

$$\hat{\mathcal{H}}_w = \frac{G}{\sqrt{2}} j_\mu^{lept} \hat{\mathcal{J}}^\mu$$

which provides the transition ME:

$$\langle f|\widehat{H_w}|i\rangle = \frac{G}{\sqrt{2}} \ell^\mu \int d^3x \, e^{-i\mathbf{qx}} \langle f|\widehat{\mathcal{J}_\mu}|i\rangle.$$

They enter the description of the cross secions for all semi-leptonic electro-weak nuclear processes.

$\overrightarrow{q}$ is the 3-momentum transfer, coming from the kinematics of the μ^--capture and given by:

$$q \equiv q_f = m_\mu - \epsilon_b + E_i - E_f$$

Formalism for the exclusive transition rates

The computation of exclusive μ^--capture rates (between an initial $|J_i\rangle$ and a final $|J_f\rangle$ nuclear state) is written in terms of the nuclear ME of the eight different tensor multipole operators (Donnelly-Walecka decomposition method) as

$$\Lambda_{i\to f} = \frac{2G^2 q_f^2}{2J_i+1} R_f \left[\left| \langle J_f \| \Phi_{1s} (\widehat{\mathcal{M}}_J - \widehat{\mathcal{L}}_J) \| J_i \rangle \right|^2 + \left| \langle J_f \| \Phi_{1s} (\widehat{\mathcal{T}}_J^{el} - \widehat{\mathcal{T}}_J^{magn}) \| J_i \rangle \right|^2 \right]$$

- Φ_{1s} represents the muon w-f (in the 1s μ^--orbit).
- R_f is a recoil factor.
- The multipole operators $\widehat{\mathcal{M}}_J$ (Coulomb), $\widehat{\mathcal{L}}_J$ (longitudinal), $\widehat{\mathcal{T}}_J^{el}$ (transverse electric) and $\widehat{\mathcal{T}}_J^{magn}$ (transverse magnetic) contain polar-vector and axial-vector parts.

Nuclear matrix elements

The eight independent multipole operators

$$\begin{aligned}
\hat{M}^{coul}_{JM}(q\mathbf{r}) &= F_1^V(q_\mu^2) M_M^J(q\mathbf{r}), \\
\hat{L}_{JM}(q\mathbf{r}) &= \frac{q_0}{q} \hat{M}^{coul}_{JM}(q\mathbf{r}), \\
\hat{T}^{el}_{JM}(q\mathbf{r}) &= \frac{q}{M_N}\left[F_1^V(q_\mu^2) {\Delta'}^J_M(q\mathbf{r}) + \frac{1}{2}\mu^V(q_\mu^2)\Sigma^J_M(q\mathbf{r})\right], \\
i\hat{T}^{mag}_{JM}(q\mathbf{r}) &= \frac{q}{M_N}\left[F_1^V(q_\mu^2) \Delta^J_M(q\mathbf{r}) - \frac{1}{2}\mu^V(q_\mu^2){\Sigma'}^J_M(q\mathbf{r})\right], \\
i\hat{M}^5_{JM}(q\mathbf{r}) &= \frac{q}{M_N}\left[F_A(q_\mu^2)\Omega^J_M(q\mathbf{r}) + \frac{1}{2}(F_A(q_\mu^2) + q_0 F_P(q_\mu^2)){\Sigma''}^J_M(q\mathbf{r})\right], \\
-i\hat{L}^5_{JM}(q\mathbf{r}) &= \left[F_A(q_\mu^2) - \frac{q^2}{2M_N}F_P(q_\mu^2)\right]{\Sigma''}^J_M(q\mathbf{r}), \\
-i\hat{T}^{el5}_{JM}(q\mathbf{r}) &= F_A(q_\mu^2){\Sigma'}^J_M(q\mathbf{r}), \\
\hat{T}^{mag5}_{JM}(q\mathbf{r}) &= F_A(q_\mu^2)\Sigma^J_M(q\mathbf{r}).
\end{aligned} \tag{1}$$

include the form factors F_X, X=1,A,P and μ^V, functions of the 4-momentum transfer q_μ^2.

Construction of nuclear ground state $|i\rangle$

To construct the nuclear ground state $|i\rangle = |g.s.\rangle$ we consider:

- A **Woods-Saxon** potential (description of strong nuclear field).
- The monopole part of the **Bonn C-D** potential (pairing interaction). Its renormalization is achieved through the two pairing parameters $g_{pair}^{p,n}$ determined by the reproducibility of the energy gaps from neighboring nuclei (3-point formula).
- The solution of BCS equations gives the quasi-particle energies and the probabilities v_k^2 and u_k^2 of each single particle level to be occupied or unoccupied.

Construction of nuclear ground state $|i\rangle$

Parameters for the renormalization of the interaction of proton pairs, g^{p}_{pair}, and neutron pairs, g^{n}_{pair}. They are fixed so as the corresponding experimental gaps, Δ_p^{exp} and Δ_n^{exp}, to be quite accurately reproduced.

Nucleus	g^{n}_{pair}	g^{p}_{pair}	Δ_n^{exp} (MeV)	Δ_n^{theor} (MeV)	Δ_p^{exp} (MeV)	Δ_p^{theor} (MeV)
^{28}Si	1.1312	1.0601	3.1428	3.1429	3.0375	3.0377
^{32}S	0.8862	0.8230	2.0978	2.0979	2.0387	2.0386
^{48}Ti	0.9259	0.9833	1.5576	1.5578	1.9112	1.9111
^{56}Fe	0.9866	0,9756	1.3626	1.3626	1.5682	1.5683
^{66}Zn	1.0059	0.9271	1.7715	1.7716	1.2815	1.2814
^{90}Zr	0.9057	0.7838	1.8567	1.8568	1.1184	1.1183

The excited nuclear states-Diagonalization of QRPA Eqs.

The excited states $|f\rangle$ are derived by solving the QRPA equations:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix} = \Omega^{\nu}_{J^{\pi}} \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix},$$

First we define new amplitudes P^m, R^m

$$(\mathcal{A}-\mathcal{B})P^m = R^m\,, \quad (\mathcal{A}+\mathcal{B})R^m = \Omega^2_m P^m$$

which are related to the X,Y through

$$X^m = \sqrt{\frac{1}{2}}(\Omega_m^{1/2}P^m + \Omega_m^{-1/2}R^m)\,, \quad Y^m = \sqrt{\frac{1}{2}}(-\Omega_m^{1/2}P^m + \Omega_m^{-1/2}R^m)$$

Finally, the diagonalization of the QRPA equations gives:

- X and Y forwards and backwards going amplitudes
- QRPA excitation energies

Parameters of Excited states

- For the renormalization of the residual interaction (Bonn C-D) we use the g_{pp} and g_{ph} parameters which are determined from the reproducibility of the low-energy experimental spectrum.
- For measuring the excitation energies of the daughter nucleus from the ground state of the initial is necessary the shifting of the entire set of QRPA eigenvalues [1, 2].

R.A. Eramzhyan, et al, *Nucl. Phys. A* **642** (1998) 428.

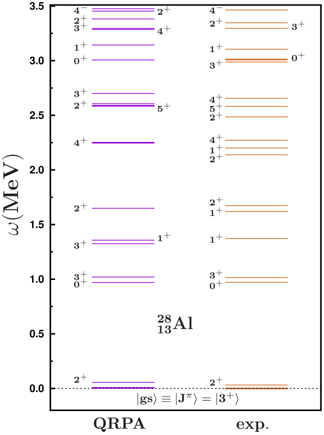
V. Rodin and A. Faessler, *Prog. Part. Nucl. Phys.* **57** (2006) 226.

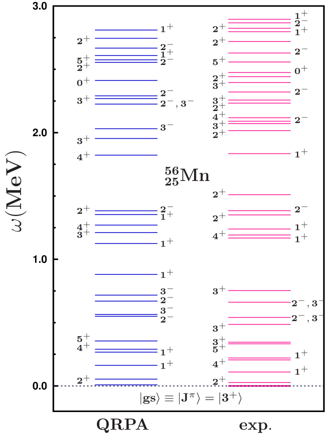
Table: The shift of the spectrum seperately of each state in MeV

Positive Parity States						Negative Parity States					
J^+	^{28}Si	^{32}S	^{48}Ti	^{56}Fe	^{66}Zn	J^-	^{28}Si	^{32}S	^{48}Ti	^{56}Fe	^{66}Zn
0^+	2.60	0.00	0.65	1.60	0.90	0^-	4.20	1.00	4.00	4.30	5.00
1^+	5.00	2.50	2.65	5.90	2.50	1^-	4.40	4.05	4.00	4.20	6.80
2^+	4.35	2.43	2.10	3.10	2.55	2^-	5.80	4.40	5.10	6.80	3.85
3^+	5.90	0.00	2.70	2.30	2.50	3^-	6.00	3.98	4.10	6.80	2.60
4^+	4.90	3.56	3.25	2.50	1.75	4^-	5.00	2.57	4.25	3.50	3.55
5^+	2.70	0.84	3.35	2.00	0.55	5^-	6.50	0.00	3.05	3.50	3.00

Low-lying QRPA Excitation Spectrum

The first step of our results is the reproducibility of the excitation spectrum. Our QRPA spectra fit well the experimental data for low lying excitations.





State-by-state calculations of exclusive μ^--capture rates

Exclusive rates

$$\Lambda_{gs\rightarrow J_f^\pi} \equiv \Lambda_{J_f^\pi} = 2G^2\langle\Phi_{1s}\rangle^2 R_f q_f^2 \quad \Big[\quad \left|\langle J_f^\pi \|(\widehat{\mathcal{M}}_J - \widehat{\mathcal{L}}_J)\|0_{gs}^+\rangle\right|^2 + \left|\langle J_f^\pi \|(\widehat{\mathcal{T}}_J^{el} - \widehat{\mathcal{T}}_J^{magn})\|0_{gs}^+\rangle\right|^2 \Big]$$

Quenching effect

$\underline{^{28}Si} \Longrightarrow g_A = 1.262$ **free nucleon coupling constant**
$\underline{^{56}Fe} \Longrightarrow g_A = 1.135$ **takes into account the small quenching effect indicated for medium-weight nuclei.**

Code

Our code provides: Seperate contributions induced by the components of muon capture operators

- polar vector
- axial vector
- overlap terms

State-by-state calculations of exclusive μ^--capture

Contribution of each multipolarity J^π

Then, we focused on the separate contributions of each multipolarity (for $J^\pi \leq 5^\pm$). In the model space chosen, we have:

$\underline{^{28}Si} \Longrightarrow$ **286 states** , $\underline{^{56}Fe} \Longrightarrow$ **488 states**

We found that the most important contributions are these of $J^\pi = 1^-$ multipolarity.

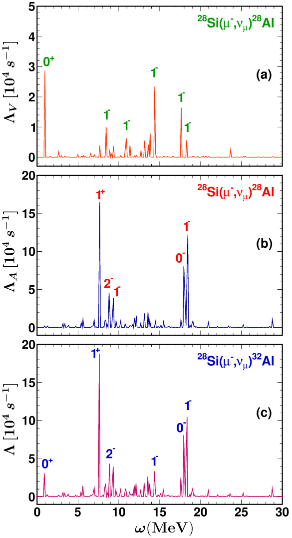
Comparison of 1^- peaks with the empirical peak of Giant Dipole Resonance

For medium-weight and heavy isotopes the empirical giant dipole resonance peak is located at energy

$$E_{IVD} = 31.2A^{-1/3} + 20.6A^{-1/6}$$

This empirical peak is in good agreement with the 1^- pronounced peak of our results.

^{28}Si isotope



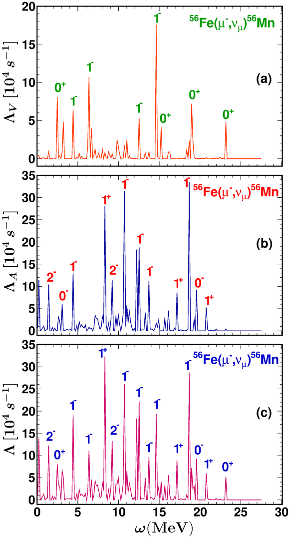
Individual contributions of the Polar-Vector, Λ_V, and Axial-Vector, Λ_A, parts to the total muon-capture rate versus the excitation energy ω.

Characteristic Peaks

- **At 7.712MeV** $\Longrightarrow 1^+_7$
- **At 18.135MeV** $\Longrightarrow 1^-_{22}$

Giannaka, Kosmas, *NPA* to be submitted.

^{56}Fe isotope



Individual contribution of the Polar-Vector, Λ_V, and Axial-Vector, Λ_A, to the total muon-capture rate as a function of the excitation energy ω.

Characteristic Peaks

- **At 8.278MeV** $\Longrightarrow 1^+_{10}$
- **At 18.716MeV** $\Longrightarrow 1^-_{38}$
- $E_{IVD} = 18.670 MeV$
  Good Agreement!!!

Partial Rates of individual multipolarities

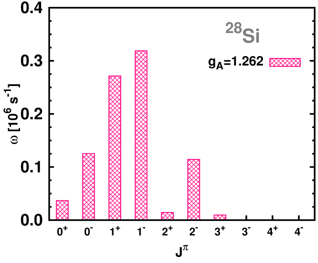
By summing over the individual contibution of the multipole states of each multipolarity we obtain the partial rates Λ_{J^π}

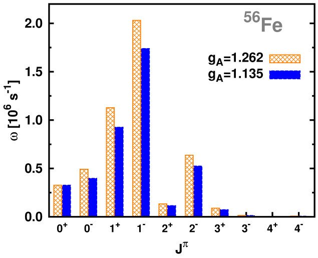
$$\Lambda_{J^\pi} = \sum_f \Lambda_{gs \to J_f^\pi} = 2G^2 \langle \Phi_{1s} \rangle^2 \Big[\sum_f q_f^2 R_f \left| \langle J_f^\pi \| (\widehat{\mathcal{M}}_J - \widehat{\mathcal{L}}_J) \| 0_{gs}^+ \rangle \right|^2 + \sum_f q_f^2 R_f \left| \langle J_f^\pi \| (\widehat{\mathcal{T}}_J^{el} - \widehat{\mathcal{T}}_J^{magn}) \| 0_{gs}^+ \rangle \right|^2 \Big]$$

f runs over all states of a given multipolarity J^π.

	^{28}Si	^{32}S	^{48}Ti	^{56}Fe	^{66}Zn	^{90}Zr
0^+	4.11	1.27	7.24	7.92	8.22	8.99
0^-	14.03	13.30	10.78	9.64	7.94	6.89
1^+	30.42	30.28	16.24	22.46	21.29	20.43
1^-	35.74	38.01	43.88	42.18	44.21	42.43
2^+	1.62	2.36	2.67	2.79	2.85	4.16
2^-	12.81	13.54	16.97	12.72	13.32	13.57
3^+	1.09	0.97	1.82	1.78	1.58	2.65
3^-	0.10	0.15	0.23	0.32	0.34	0.52
4^+	0.01	0.01	0.01	0.01	0.01	0.03
4^-	0.06	0.10	0.14	0.16	0.23	0.30

The percentage of each multipolarity into the total muon-capture rate.





Partial transition rates Λ_{J^π} (for $J^\pi \leq 4^\pm$) of ^{28}Si and ^{56}Fe isotopes. The most important partial transition rates (as expected) come from the $J^\pi = 1^-$ and $J^\pi = 1^+$ low lying excitations.

^{28}Si

- $J^\pi = 1^-$ **contribute** 36%
- $J^\pi = 1^+$ **contribute** 30%

^{56}Fe

- $J^\pi = 1^-$ **contribute** 42%
- $J^\pi = 1^+$ **contribute** 22%

There are no similar detailed calculations to compare with.

Giannaka, Kosmas, *NPA* to be submitted.

Total μ^--capture Results

The total muon capture rates are obtained by summing over all partial multipole transition rates.

$$\Lambda_{tot} = \sum_{J^\pi} \Lambda_{J^\pi} = \sum_{J^\pi} \sum_{f} \Lambda_{J^\pi_f}$$

Total Muon-capture rates $\Lambda_{tot}(\times 10^6)s^{-1}$

	Present pn-QRPA Calculations				Experiment	Other theoretical Methods	
Nucleus	Λ^{V}_{tot}	Λ^{A}_{tot}	Λ^{VA}_{tot}	Λ_{tot}	Λ^{exp}_{tot}	Λ^{theor}_{tot} [1]	Λ^{theor}_{tot} [2]
^{28}Si	0.150	0.751	-0.009	0.892	0.871	0.823	0.789
^{32}S	0.204	1.078	-0.017	1.265	1.352	1.269	1.485
^{48}Ti	0.628	1.902	-0.081	2.447	2.590	2.214	2.544
^{56}Fe	1.075	3.179	-0.129	4.125	4.411	4.457	4.723
^{66}Zn	1.651	4.487	-0.204	5.934	5.809	4.976	5.809
^{90}Zr	2.679	7.310	-0.357	9.631	9.350	8.974	9.874

Zinner, Langanke, Vogel, *PRC* **74** (2006) 024326.

T. Marketin, et al, *PRC* **79** (2009) 054323.

Summary and Conclusions

- Relying on an advantageous numerical code constructed by our group, we calculated all multipole transition ME entering the exclusive μ^--capture rates.
- The required nuclear wave functions obtained within the context of the pn-QRPA using realistic two-body forces.
- Results for exclusive, partial and total muon capture rates are obtained using a free nucleon coupling constant $g_A = 1.262$ for light nuclei and a quenched value of $g_A = 1.135$ for medium-weight nuclei.
- As pn-QRPA provides an accurate description of all semileptonic processes, we have already extent our method in electron capture process.
- Next step $\Longrightarrow$ To extent this method to other semi-leptonic nuclear processes like beta-decay and charged current neutrino nucleus processes and apply our method in nuclear applications and astrophysics and netrino nucleosynthesis.

Collaborations

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- **University of Tuebingen, Germany**: K. Kokkotas
- **University of Darmstant, Germany**: Group of Langanke

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