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"Realistic nuclear structure calculations for orbital e-capture by nuclei"

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Outline

- Introduction
- Theoretical backround
- Results and discussion
- Summary and Conclusions

Weak interaction processes in nuclei

- Elementary weak processes:

$$n \to p + e^- + \bar{\nu}_e$$

$$p \to n + e^+ + \nu_e$$

- Weak one body processes in nuclei

i) $\beta^\pm$ decay modes

$$\left\{ \begin{array}{l} {}^A_Z X \to {}^A_{Z-1} Y + e^+ + \bar{\nu}_e \\ {}^A_Z X \to {}^A_{Z+1} Y + e^- + \nu_e \end{array} \right.$$

ii) lepton (e,μ,τ) capture by nuclei

e-capture

$$(A, Z) + e^- \to (A, Z-1)^* + \nu_e$$

μ-capture

$$(A, Z) + \mu \to (A, Z-1)^* + \nu_\mu$$

iii) ν-nucleus induced reactions

$$\left\{ \begin{array}{lll} \nu_l + (A, Z) & \longrightarrow & (A, Z+1)^* + l^- \\ \overline{\nu}_l + (A, Z) & \longrightarrow & (A, Z-1)^* + l^+ \\ \nu + (A, Z) & \longrightarrow & (A, Z)^* + \nu' \\ \overline{\nu} + (A, Z) & \longrightarrow & (A, Z)^* + \overline{\nu}' \end{array} \right.$$

Role of e-capture in astro-physics

- e-capture process plays important role in core collapse SNe. The collapse starts mainly due to

$$^{56}Fe + e^- \rightarrow ^{56}Mn + \nu_e$$

Experimentally is well studied

Theoretically is investigated using various nuclear models

i. Fermi gas models
ii. Shell model
iii. Random Phase Approximation (RPA), Quasi-particle Random Phase Approximation (QRPA)

- e-capture is particle conjugate process of CC v_e-nucleus reaction (initial and final particles are interchanged)

Weak interaction Hamiltonian

The hamiltonian of weak interaction is written in current-current form as:

$$\hat{\mathcal{H}}_w = \frac{G\cos\theta_c}{\sqrt{2}} j_\mu^{lept}(\mathrm{x})\hat{\mathcal{J}}_\mu(\mathrm{x})$$

"current-current interaction"

G=weak coupling constant

- Leptonic Current → j_μ^{lept}
- Hadronic Current → $\mathcal{J}_\mu$

The hadronic current (nucleon level)

Focusing on the hadronic current, the most important from a nuclear theory point of view,we write

$$\hat{\mathcal{J}}_\mu = g_V \hat{J}^V_\mu - g_A \hat{J}^A_\mu$$

polar-vector axial-vector

These components are written (nucleon level)

$$< p' \mid \hat{J}^{\pm}_{\mu 5}(0) \mid p >= \bar{u}(p')[F_A \gamma_5 \gamma_\mu - i F_P \gamma_5 q_\mu]\tau_\pm u(p)$$
$$< p' \mid \hat{J}^{\pm}_{\mu}(0) \mid p >= \bar{u}(p')[F^V_1 \gamma_\mu + F^V_2 \sigma_{\mu\nu} q^\nu]\tau_\pm u(p)$$

Where F_i the hadronic Form Factors, function of the,q_μ^2, 4-momentum transfer

$$q^2_\mu = -4\varepsilon_i \varepsilon_f \sin^2 \frac{\theta}{2}$$

$$q = |\mathrm{q}| = \sqrt{\omega^2 + 4\varepsilon_i \varepsilon_f sin^2 \frac{\Theta}{2}}$$

Form Factors

- Isovector form factors

$$F_1^V(q^2) = \left(1 - \frac{q^2}{4M}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}G_M^V(q^2)\right]$$

$$F_2^V(q^2) = \left(1 - \frac{q^2}{4M}\right)^{-1}\left[G_M^V(q^2) - G_E^V(q^2)\right]$$

- Sach's electric and magnetic form factors

$$G_E^V(q^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2}\left[1 - \frac{q^2\mu_n}{4M^2}\right], \qquad G_M^V(q^2) = \frac{1 + \mu_p - \mu_n}{\left(1 - \frac{q^2}{M_V^2}\right)^2}$$

- Axial Vector form factor

$$F_A^V(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

- Pseudoscalar form factor

$$F_p^V(q^2) = \frac{2MF_A^V(q^2)}{M_\pi^2 - q^2}$$

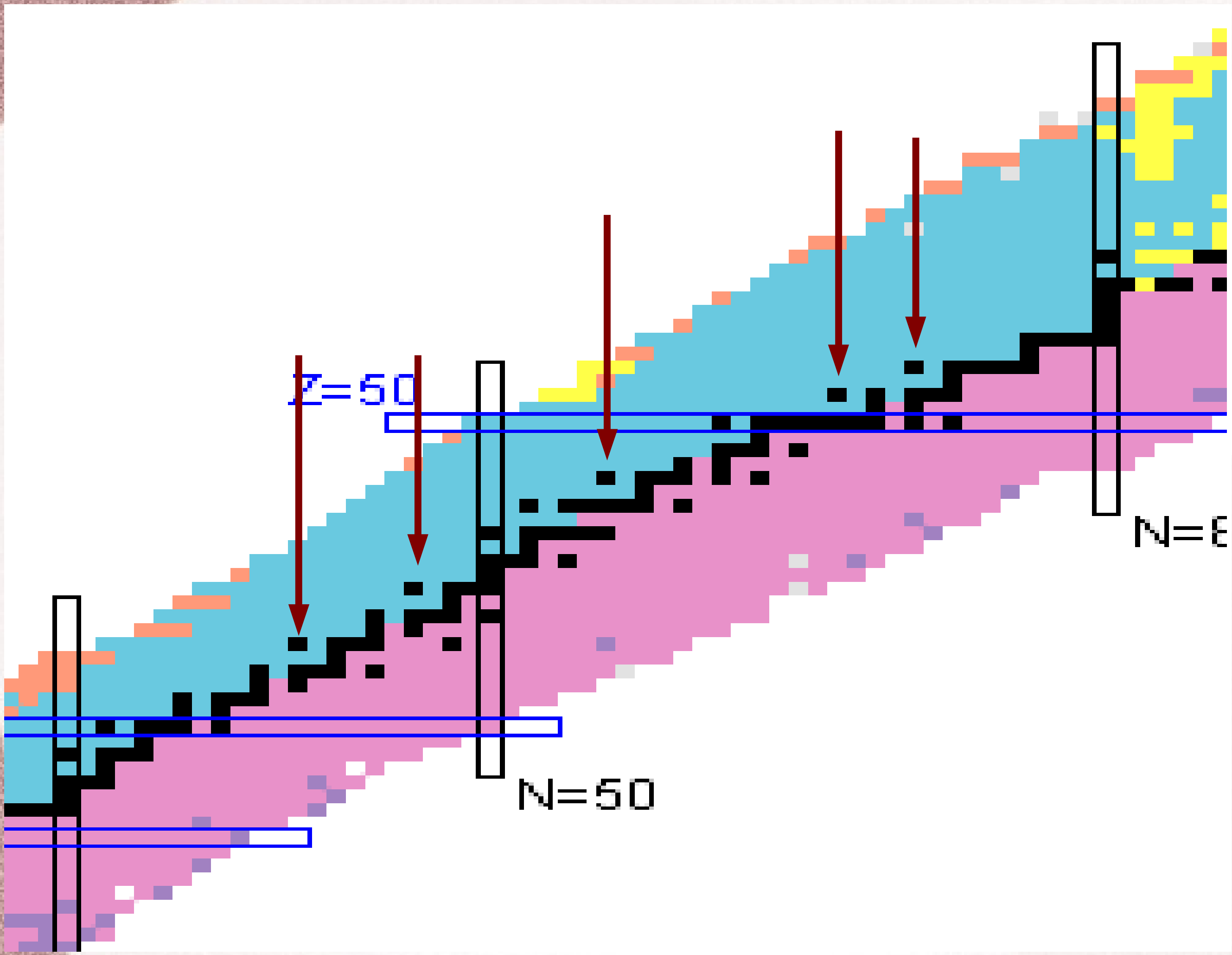
Results and Discussion

1)We have chosen to study the e-capture by the iron group peaked nuclei: ^{96}Ru, ^{98}Ru, ^{106}Cd, ^{108}Cd, ^{84}Sr, ^{78}Kr, ^{74}Se, ^{102}Pd, ^{92}Mo, ^{94}Mo ⇨ The lightest stable isotope of isotopes chain belonging to the same chemical element, (Z= constant).They lie close to the proton drip line
These nuclei are important from an astrophysical point of view in:
C. Frohlich,G. Martinez-Pinedo,K.Langanke, PRL 96. 142502 (2006)

(a)Explosive nucleosynthesis (the main subject of my PhD Thesis)
(b)Stellar nucleosynthesis and evolution of chemical elements in Galaxies

Important isotopes for e⁻-capture

The isotopes studied here are stable



Periodic Table

QRPA calculations

We are going to do detailed Nuclear structure calculations for e^--capture by using the QRPA method

From a nuclear structure calculations point of view this study includes:

- Construct the energy spectrum (QRPA energies)
- Make various checks for the QRPA nuclear wave functions
- Perform detailed calculations for the corresponding cross sections

Up to now we have computed:
The ground state of the studied isotopes
The excited states of the ^{96}Ru and ^{102}Pd

We expect to receive results for cross sections soon (to be presented in MEDEX 2013, topic “e-capture revisited”)

The Nuclear Hamiltonian Interaction

a)Mean field (central potential – Woods Saxon)
b)Residual interactions Bonn C-D (one meson exchange potential)

Renormalization of the interaction for the explicit nuclear isotope studied

Construction of the nuclear ground state |i>

1)Ground state is obtained in the context of BCS

- Solution of BCS Equations

$$\upsilon_k^2 = \frac{1}{2}\left[1 - \frac{\tilde{\varepsilon}_k}{\sqrt{\tilde{\varepsilon}_k^2 + \Delta_k^2}}\right] \qquad u_k^2 = \frac{1}{2}\left[1 + \frac{\tilde{\varepsilon}_k}{\sqrt{\tilde{\varepsilon}_k^2 + \Delta_k^2}}\right]$$

$$\Delta_k = -\sum_{k'>0} \bar{\upsilon}_{k\bar{k}_{k'}\bar{k'}} u'_k \upsilon'_k$$

- Determination of pairing parameters $g_{pair}^{p(n)}$

The parameters for the interaction of proton and neutron pairs g_{pair}^{p} and neutron pairs g_{pair}^{n} are determined by adjusting the empirical energy-gaps from neighboring nuclei. This is the three-point formula

$$\Delta_n^{exp} = -\frac{1}{4}\{S_n[(N-1),Z)] - 2S_n[(N,Z)] + S_n[(N+1),Z)]\}$$

$$\Delta_p^{exp} = -\frac{1}{4}\{S_p[(N,Z-1)] - 2S_p[(N,Z)] + S_p[(N,Z+1)]\}$$

Determination of nuclear parameters

	Abundance(%)	b (h.o.)	g^n_{pair}	g^p_{pair}	S_n (MeV)	S_p (MeV)	Δ^{exp}_n (MeV)	Δ^{theor}_n (MeV)	Δ^{exp}_p (MeV)	Δ^{theor}_p (MeV)
^{96}Ru	5,540	2,158	0,987	0,835	10,694	7,344	1,0824	1,0821	1,4955	1,4954
^{98}Ru	1,870	2,165	0,978	0,889	10,183	8,293	1,1971	1,1975	1,5567	1,5535
^{106}Cd	1,250	2,190	0,886	0,872	10,874	7,353	1,3492	1,3497	1,5057	1,5057
^{108}Cd	0,890	2,196	0,927	0,965	10,339	8,140	1,3567	1,3574	1,4917	1,4924
^{84}Sr	0,560	2,116	1,027	0,861	11,923	8,867	1,6137	1,6142	1,8685	1,8698
^{78}Kr	0,350	2,094	0,979	0,812	12,081	8,234	1,6360	1,6354	1,8177	1,8172
^{74}Se	0,890	2,078	0,963	0,823	12,066	8,545	1,9279	1,9272	1,8037	1,8036
^{102}Pd	1,020	2,178	0,978	0,958	10,568	7,806	1,3094	1,3085	1,4947	1,4938
^{92}Mo	14,53	2,145	1,135	0,637	12,673	7,457	1,7923	1,7920	1,4171	1,4162
^{94}Mo	9,150	2,152	0,908	0,870	9,6780	8,490	0,9793	0,9781	1,5105	1,5094

Construction of the excited QRPA states

Excited states |f> are obtained in the context of QRPA

- Solution of QRPA Equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = \Omega^m_{J^\pi} \begin{pmatrix} X^m \\ Y^m \end{pmatrix}$$

where X,Y are the amplitudes for forward and backward scattering

- Determination of strength parameters g_{pp} (particle-particle) and g_{ph} (particle-hole)

In the Donelly-Walecka method the solution of the QRPA Equations are obtained separetely for each multipole set of states.

Nuclear Spectrum

The determination of the g_{pp} and g_{ph} parameters is done separately for each multipole set of states

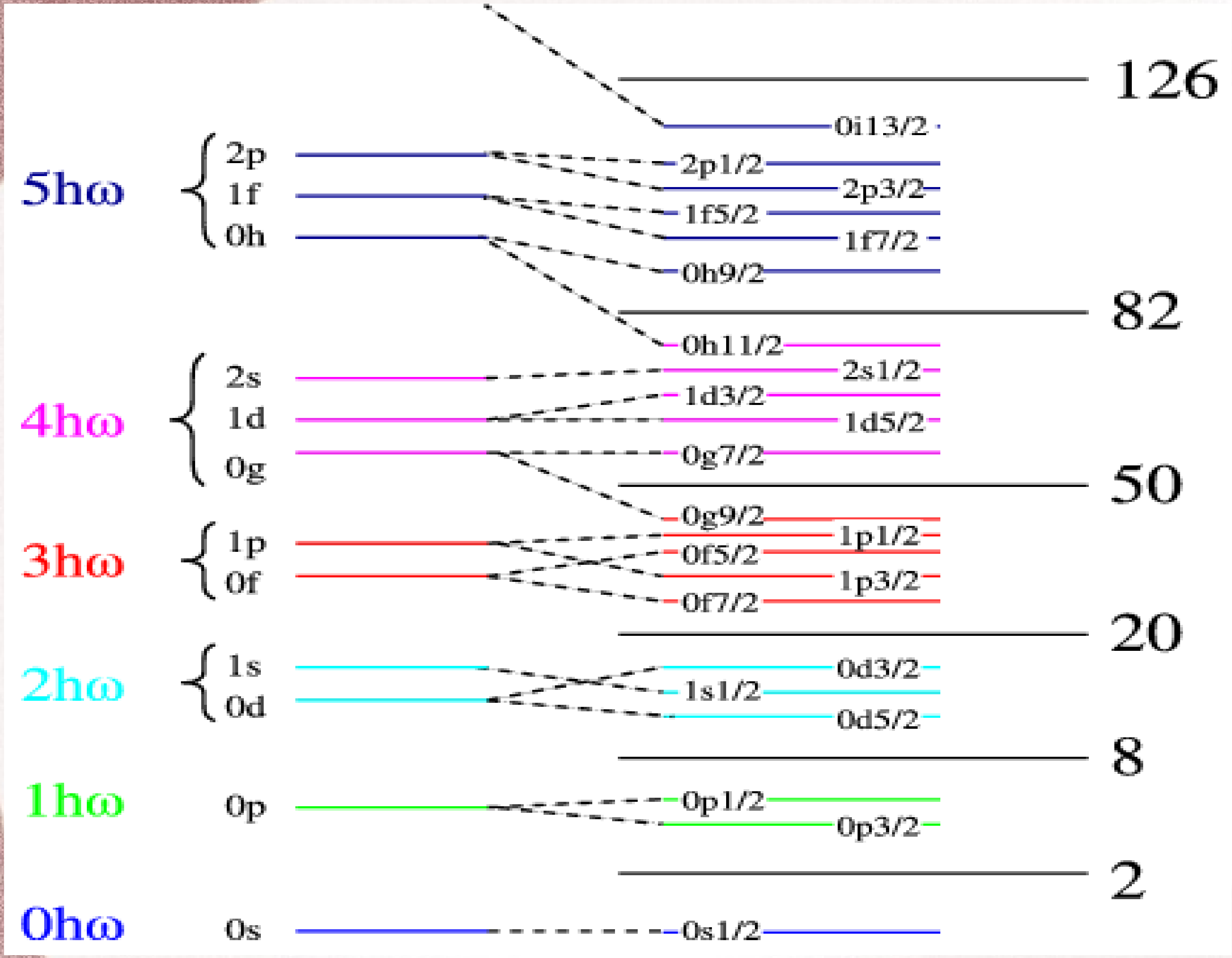
.

	^{96}Ru			^{102}Pd	
	g_{ph}	g_{pp}		g_{ph}	g_{pp}
0+	0.403	0.781	0+	0.377	0.907
2+	0.579	1.189	2+	0.671	1.350
4+	0.905	0.546	4+	1.040	0.322
6+	1.085	1.195	6+	1.108	0.247
3+	1.000	1.000	3+	1.000	1.000
1+	1.000	1.000	5+	1.000	1.000

The model space used for ^{96}Ru consists of 14 active levels (core ^{16}O)
Major shells with N= $2\hbar\omega, 3\hbar\omega, 4\hbar\omega$ and levels $0h_{11/2}, 0h_{9/2}$

Whereas for ^{102}Pd consists of 11 active levels (core ^{40}Ca)
Major shells with N= $3\hbar\omega, 4\hbar\omega$ and levels $0h_{11/2}, 0h_{9/2}$

Nuclear Shell model



Summary and Conclusions

- For a set of nuclear isotopes we started to study e- capture processes.This set of nuclei is important for explosive nucleosynthesis and chemical element evolution.

- We have constructed the ground state of the nuclei using BCS method and the excited states using QRPA

- We expect to receive e-capture cross sections soon

Don't forget to mention that:

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Co- financed by Greece and the European Union

Thank you for your attention!!!

Form Factors

k	$a^{M}_{p,k}$	$b^{M}_{p,k}$	$a^{M}_{n,k}$	$b^{M}_{n,k}$	$a^{E}_{p,k}$	$b^{E}_{p,k}$
1	1.09	12.31	8.28	21.30	-0.19	11.12
2	-	25.57	-	77	-	15.16
3	-	30.61	-	238	-	21.25

Compact expressions for the 7-basic reduced ME

For H.O. Bases wave functions all basic reduced ME take the compact forms

$$\langle j_1||T^J||j_2\rangle = e^{-y}y^{\beta/2}\prod(y) = e^{-y}y^{\beta/2}\sum_{\mu=0}^{n_{max}}\mathcal{P}_\mu^J\, y^\mu$$

$$\prod(y) = \sum_{\mu=0}^{n_{max}}\mathcal{P}_\mu^J\, y^\mu, \quad y = \frac{q^2b^2}{4}$$

$$n_{max} = (N_1 + N_2 - \beta)/2\,.$$

Advantages of the above Formalism :

(i) The coefficients P^J are calculated once (reduction of computer time)

(ii) They can be used for phenomenological description of ME

(iii) They are useful for other bases sets (expansion in HO wave-functions)

Definition of the coefficients P_μ^J

Operator	β	$\mathcal{P}_\mu^J, 0 \leqslant \mu \leqslant n_{\max}$
$T_1^J = M^J = \delta_{LJ}\, j_L(\rho) Y_M^L(\hat{r})$	J	$E_{1,\mu}^J = (l_1 J l_2) \mathcal{U}_{JS_1}^J \varepsilon_\mu^J (n_1 l_1 n_2 l_2)$
$T_2^J = \Sigma^J = \mathbf{M}_M^{JJ} \cdot \sigma$	J	$E_{2,\mu}^J = (l_1 J l_2) \mathcal{U}_{JS_2}^J \varepsilon_\mu^J (n_1 l_1 n_2 l_2)$
$T_3^J = \Sigma'^J = -i[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})] \cdot \sigma$	$J-1$	$(J+1)^{1/2} E_{2,\mu}^{J-1} - J^{1/2} E_{2,\mu-1}^{J+1}$
$T_4^J = \Sigma''^J = [\frac{1}{q}\nabla M_M^J(q\mathbf{r})] \cdot \sigma$	$J-1$	$J^{1/2} E_{2,\mu}^{J-1} + (J+1)^{1/2} E_{2,\mu-1}^{J+1}$
$T_5^J = \Delta^J = \mathbf{M}_M^{JJ}(q\mathbf{r}) \cdot \frac{1}{q}\nabla$	$J-1$	$E_{3,\mu}^L = \mathcal{A}_L^- \zeta_\mu^-(L) + \mathcal{A}_L^+ \zeta_\mu^+(L)$
$T_6^J = \Delta'^J = -i[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})]$	$J-2$	$(J+1)^{1/2} E_{3,\mu}^{J-1} - J^{1/2} E_{3,\mu-1}^{J+1}$
$T_7^J = \Omega^J = M_M^J(q\mathbf{r})\sigma \cdot \frac{1}{q}\nabla$	J	$E_{4,\mu}^J = \mathcal{B}_L^- \zeta_\mu^-(L) + \mathcal{B}_L^+ \zeta_\mu^+(L)$
$\Omega'^J = \Omega_M^J + \frac{1}{2}\Sigma''^J_M$	$J-1$	$E_{4,\mu}^J + \frac{1}{2}\{J^{1/2} E_{2,\mu}^{J-1} + (J+1)^{1/2} E_{2,\mu-1}^{J+1}\}$

	Abundance(%)	b (h.o.)	g^{n}_{pair}	g^{p}_{pair}	S_n (MeV)	S_p (MeV)	Δ^{exp}_{n} (MeV)	Δ^{theor}_{n} (MeV)	Δ^{exp}_{p} (MeV)	Δ^{theor}_{p} (MeV)
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^{106}Cd	1,250	2,190	0,886	0,872	10,874	7,353	1,3492	1,3497	1,5057	1,5057
^{108}Cd	0,890	2,196	0,927	0,965	10,339	8,140	1,3567	1,3574	1,4917	1,4924
^{84}Sr	0,560	2,116	1,027	0,861	11,923	8,867	1,6137	1,6142	1,8685	1,8698
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^{102}Pd	1,020	2,178	0,978	0,958	10,568	7,806	1,3094	1,3085	1,4947	1,4938
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^{94}Mo	9,150	2,152	0,908	0,870	9,6780	8,490	0,9793	0,9781	1,5105	1,5094

The nuclear semi-empirical mass formula

$$M(Z,A)c^2 = ZM_pc^2 + (A-Z)M_nc^2 - B(Z,A)$$

Where

$$B(Z,A) = a_vA - a_sA^{2/3} - a_cZ^2/A^{1/3} - a_A(A-2Z)^2/A \begin{cases} -a_p/A^{1/2} \\ +0 \\ +a_p/A^{1/2} \end{cases}$$

Asymmetry Term

Surface energy Term

Coulomb Term

Asymmetry Term

Pairing Term

- Ενεργειακές Περιοχές Νετρίνων:

i) Περιοχή χαμηλής ενέργειας $E_\nu \leq 20$ MeV

1)Ηλιακά νετρίνα

2)Χαμηλής ενέργειας Supernova νετρίνα

ii) Περιοχή μέσης ενέργειας 20MeV $\leq E_\nu \leq$ 50 MeV

1)Υψηλής ενέργειας Supernova νετρίνα

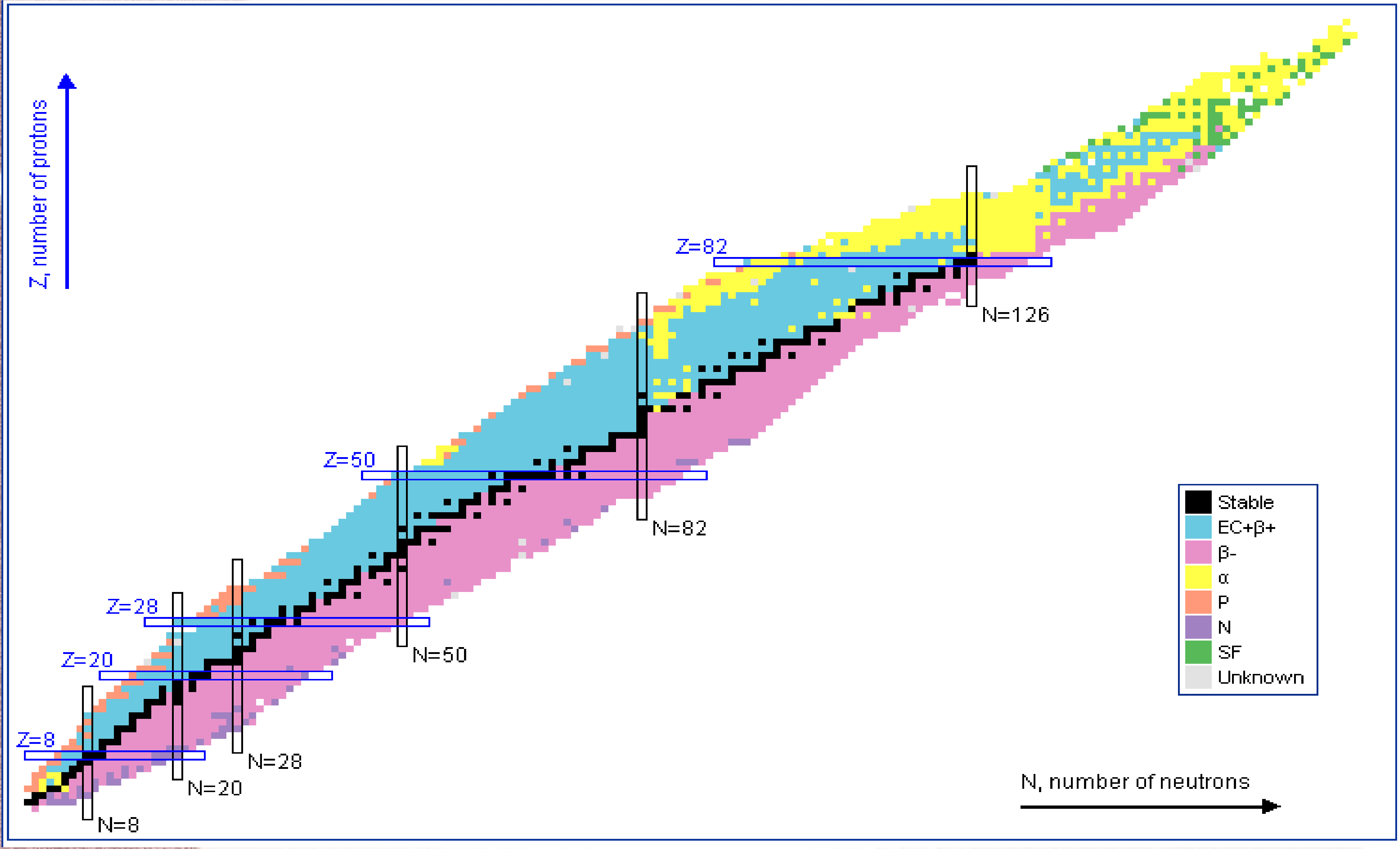
iii) Περιοχή υψηλής ενέργειας 50MeV $\leq E_\nu \leq$ 1-2 GeV

1) Ηλιακών Εκλάμψεων νετρίνα

2) Αστροφυσικά νετρίνα

	Abundance (%)	S_p (MeV)	S_n (MeV)	Δp^{exp} (MeV)	Δn^{exp} (MeV)	Δp^{th} (MeV)	Δn^{th} (MeV)		
^{96}Ru	5.54	7,344	10,694	1,495	1,082				
^{96}Ru	1.87	8,293	10,183	1,557	1,197				
^{106}Cd	1.25	7,353	10,874	1,506	1,349				
^{108}Cd	0.89	8,140	10,339	1,492	1,357				
^{84}Sr	0.56	8,867	11,923	1,868	1,614				
^{78}Kr	0.355	8,234	12,081	1,818	1,636				
^{74}Se	0.89	8,545	12,066	1,804	1,928				
^{102}Pd	1.02	7,806	10,568	1,495	1,309	1,4938	1,308		
^{92}Mo	14.53								
^{94}Mo	9.15								

Περιοδικός Πίνακας



Basic Operators (Normal or Abnormal Parity)

In the D.-W. method we need the ME of 7 basic definite parity operators

$$T_1^{JM} \equiv M_M^J(q\mathbf{r}) = \delta_{LJ}\, j_L(\rho) Y_M^L(\hat{r}),$$

$$T_2^{JM} \equiv \Sigma_M^J(q\mathbf{r}) = \mathbf{M}_M^{JJ} \cdot \boldsymbol{\sigma},$$

$$T_3^{JM} \equiv \Sigma'^J_M(q\mathbf{r}) = -i\left\{\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})\right\} \cdot \boldsymbol{\sigma},$$

$$T_4^{JM} \equiv \Sigma''^J_M(q\mathbf{r}) = \{\frac{1}{q}\nabla M_M^J(q\mathbf{r})\} \cdot \boldsymbol{\sigma},$$

$$T_5^{JM} \equiv \Delta_M^J(q\mathbf{r}) = \mathbf{M}_M^{JJ}(q\mathbf{r}) \cdot \frac{1}{q}\nabla,$$

$$T_6^{JM} \equiv \Delta'^J_M(q\mathbf{r}) = -i\{\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})\} \cdot \nabla,$$

$$T_7^{JM} \equiv \Omega_M^J(q\mathbf{r}) = M_M^J(q\mathbf{r})\boldsymbol{\sigma} \cdot \frac{1}{q}\nabla.$$

Determination of nuclear parameters

	Abundance (%)	b (h.o.)	g_{pair}^{n}	g_{pair}^{p}	S_n (MeV)	S_p (MeV)	Δ_n^{exp} (MeV)	Δ_n^{theor} (MeV)	Δ_p^{exp} (MeV)	Δ_p^{theor} (MeV)
^{96}Ru	5,540	2.158	0,987	0,835	10,694	7,344	1,0824	1,0821	1,4955	1,4954
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