1st One Day Workshop on
New Aspects and Perspectives in Nuclear Physics
8th of September, 2012, Ioannina, Greece

“Flavour changing neutral-current processes in nuclei”

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outline

- **Neutrino-nucleus interactions**
  1. Standard Model ν-nucleus processes
  2. Exotic ν-nucleus processes (FCNC)

$$\nu_\alpha + (A,Z) \rightarrow \nu_\beta + (A,Z)^*$$

- **LFV processes**
  1. Other LFV processes in nuclei (μ^--e^- conversion)
  2. The Seesaw mechanism in LFV
  3. The hadronic current for FCNC

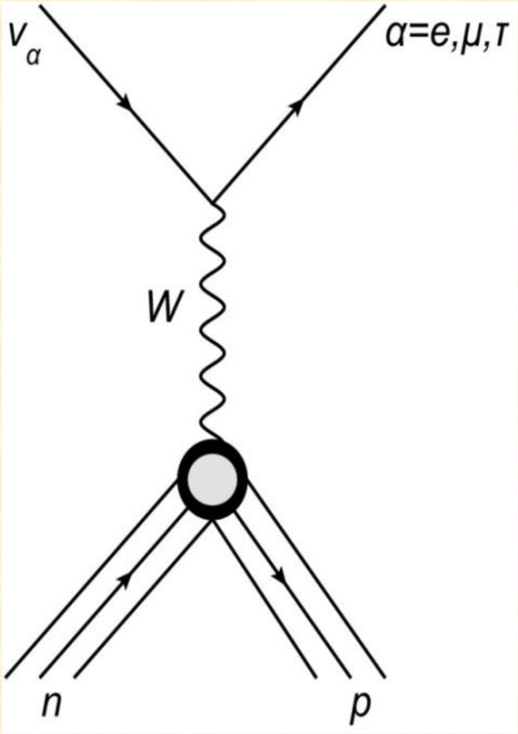
- **Theoretical Nuclear physics**
  1. The basic nuclear operators
  2. Nuclear transition matrix elements
  3. Results
  4. FCNC cross-sections

- **Outlook**

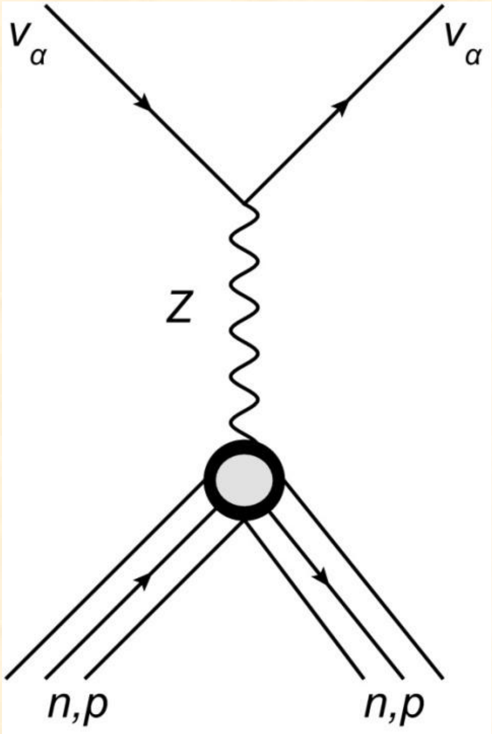
ν-nucleus weak interactions

i. **Charged-current (CC) processes (mediated by *W*-bozon)**
ii. **Neutral -current (NC) processes (mediated by *Z*-bozon)**

CC process



NC process



Exotic ν-nucleus processes

Our aim is to systematically study the ν-nucleus flavour changing neutral-current (FCNC) processes:

(one-body processes)

$$\nu_\alpha + (A,Z) \rightarrow \nu_\beta + (A,Z)^*$$

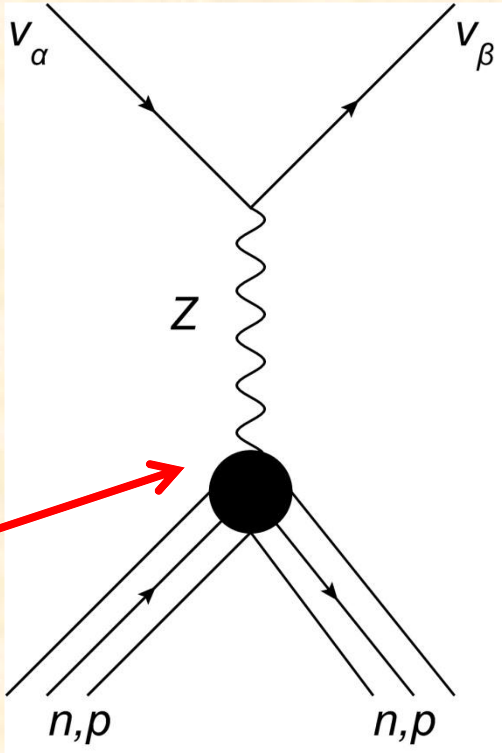
$$\tilde{\nu}_\alpha + (A,Z) \rightarrow \tilde{\nu}_\beta + (A,Z)^*$$

$\alpha, \beta = e, \mu, \tau$ **flavour indices**

$\alpha \neq \beta$

non-standard physics

Exotic neutral current ν-Nucleus reaction



Other lepton LFV processes in nuclei

In addition to $_{A}N_{Z}\,(\nu_{\alpha},\nu_{\beta})\,_{A}N_{Z}^{*}$

there are also other LFV processes in nuclei, involving charged particles, such as:

1) $\mu_b^- + (A,Z) \rightarrow e^- + (A,Z)^*$ *muon to electron conversion* (one-body process)

a) violates the lepton flavour quantum numbers L_μ and L_e but preserves the total lepton number L

b) cannot distinguish between Dirac and Majorana neutrinos

2) $\mu_b^- + (A,Z) \rightarrow e^+ + (A,Z-2)^*$ *muon to positron conversion* (two-body process)

a) violates the conservation of the total lepton number L as well as the lepton flavour quantum numbers, L_e and L_μ

b) only Majorana neutrinos permitted

[T.S.K., A. Faessler and Vergados J.Phys.G23(1997)693; *T.S. Kosmas*, NPA 683 (2001) 443]

Comparison of μ^--e^- conversion and exotic *ν-nucleus* processes

- μ^--e^- conversion has been extensively studied both experimentally and theoretically

[*T.S. Kosmas*, NPA 683 (2001) 443; *Deppisch, Kosmas, Walle*, NPB 752 (2006) 80]

Experimental limits of the branching ratio:

$$R = \frac{\Gamma_{(\mu^- \to e^-)}}{\Gamma_{(\mu^- \to \text{capture})}} \leq 5.0 \times 10^{-13}$$

(PSI experiment)

Project X, Fermilab (2016-) Expected limit: 10^{-18}

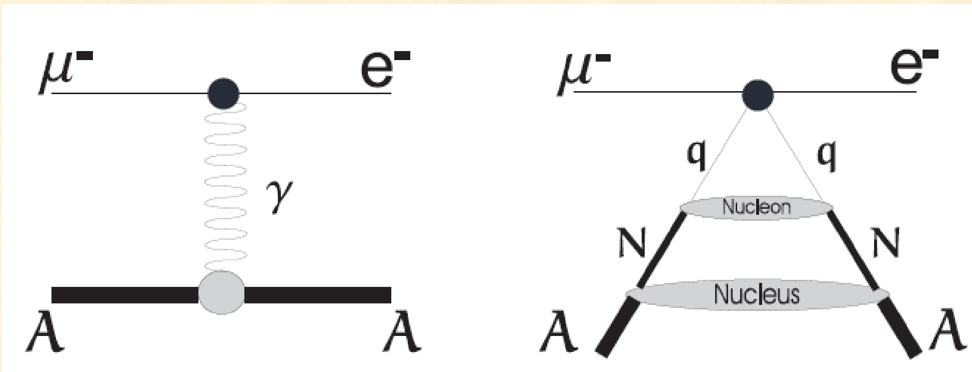
- FCNC *ν-nucleus* reactions and μ^--e^- conversion can be described within the same particle physics model, ***e.g. The Seesaw model***

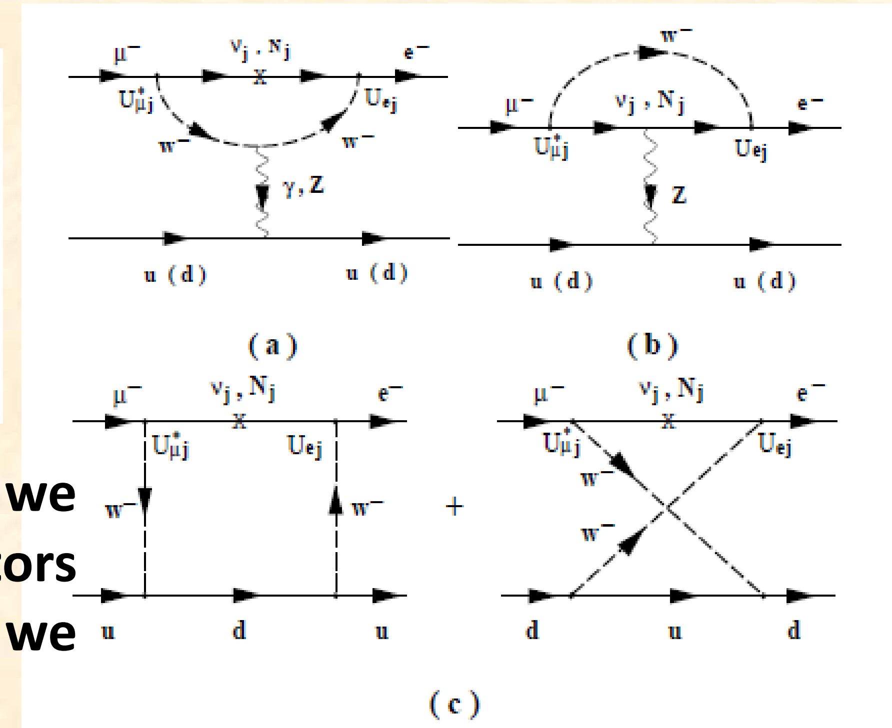
From an Astrophysical point of view the latter have been effectively studied [Amanik, Fuller]

[*Amanik, Fuller*, PRD 75 (2007) 083008]

μ^--e^- conversion within the Seesaw mechanism

The **LFV** arises from penguin photon and *Z* exchange as well as box diagrams with *W* exchange





Following [*Deppisch, Kosmas, Walle*], we will construct the corresponding operators for the exotic FCNC ν-nucleus reactions we study at:

1) Quark-level (in the context of Seesaw model and BSM models)
2) Nucleon-level (nucleon isospin operators)
3) Nuclear-level (Donnelly-Walecka method)

[T.S. Kosmas, PPNP 48(2002) 307; *Deppisch, Kosmas, Walle*, NPB 752 (2006) 80]

Nucleon-level hadronic current for ν-nucleus processes

- **The effective nucleon level Lagrangian, in terms of the effective nucleon fields and nucleon isospin operators, takes the form**

$$\mathcal{L}^{N}_{eff} = G_a\Big[\sum_{A,B} j^A_\mu \left(\alpha^{(0)}_{AB} J^{B\mu}_{(0)} + \alpha^{(3)}_{AB} J^{B\mu}_{(3)}\right) + \sum_{C,D} j^C \left(\alpha^{(0)}_{CD} J^{D}_{(0)} + \alpha^{(3)}_{CD} J^{D}_{(3)}\right) + \left(j_{\mu\nu}(\alpha^{(0)}_T J^{\mu\nu}_{(0)} + \alpha^{(3)}_T J^{\mu\nu}_{(3)}\right)\Big], \qquad a = \mathrm{ph}, \mathrm{nph}.$$

- **The isoscalar $J_{(0)}$ and isovector $J_{(3)}$ nucleon currents are defined as**

$$J^{V\mu}_{(k)} = \bar{N}\gamma^\mu \tau_k N, \quad J^{A\mu}_{(k)} = \bar{N}\gamma^\mu\gamma_5 \tau_k N, \quad J^{S}_{(k)} = \bar{N}\tau_k N, \quad J^{P}_{(k)} = \bar{N}\gamma_5 \tau_k N, \quad J^{\mu\nu}_{(k)} = \bar{N}\sigma^{\mu\nu}\tau_k N$$

$$A, B = \{A, V\} \qquad C, D = \{S, P\} \qquad N = \{p, n\}$$

- **Each component separately treated**
- **The pseudoscalar and tensor nucleon current components can be neglected**

[T.S. Kosmas, PPNP 48(2002) 307; *Deppisch, Kosmas, Walle*, NPB 752 (2006) 80]

Effective Interaction Hamiltonian

At nuclear level, the effective interaction Hamiltonian has the well-known current-current form:

$$\hat{H}_{eff} = \int d^3x \hat{\mathcal{H}}_{eff}(\mathbf{x}) = \frac{G}{\sqrt{2}} \int d^3\mathbf{x} \hat{\mathcal{J}}_\mu(\mathbf{x}) j^\mu_{lept}(\mathbf{x}),$$

Matrix Elements between initial and final Nuclear states are needed for partial transition rates :

$$\langle f|\hat{H}_{eff}|i\rangle = \frac{G}{\sqrt{2}} l^\mu \int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}} \langle f_{nucl}|\hat{\mathcal{J}}_\mu(\mathbf{x})|i_{nucl}\rangle$$

$$\hat{\mathcal{J}}^\mu(\mathbf{x}) = \mathbf{J}^{\mathbf{V},\mu}(\mathbf{x}) - \mathbf{J}^{\mathbf{A},\mu}(\mathbf{x})$$

(hadronic current, V-A theory)

$$\langle l_f|j^{\text{lept}}_\mu|l_i\rangle = l_\mu\, e^{-i\mathbf{q}\cdot\mathbf{x}}$$

(leptonic current ME)

$$q = \kappa_2 \mp \kappa_1 = p - p'$$

(momentum transfer)

The seven basic nuclear operators

$$\langle n_1(l_1 1/2) j_1 || T_i^J || n_2(l_2 1/2) j_2 \rangle \equiv \langle j_1 || T_i^J || j_2 \rangle, \quad i = 1, 2, \ldots, 7$$

From a nuclear physics point of view, electroweak processes in nuclei are described (to a rather good approximation) through the evaluation of the ***seven basic nuclear operators***

The multipole expansion of the hadronic current leads to the seven basic nuclear operators, written in terms of the ***spherical Bessel*** **(or vector) function and of the** ***spherical harmonics*** **(or vector)**

CVC theory implies that only seven irreducible tensor operators are independent

$$
\begin{aligned}
T_1^{JM} &\equiv M_M^J(q\mathbf{r}) = \delta_{LJ}\, j_L(\rho) Y_M^L(\hat{r}), \\
T_2^{JM} &\equiv \Sigma_M^J(q\mathbf{r}) = \mathbf{M}_M^{JJ} \cdot \sigma, \\
T_3^{JM} &\equiv \Sigma'^J_M(q\mathbf{r}) = -i\left[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})\right] \cdot \sigma, \\
T_4^{JM} &\equiv \Sigma''^J_M(q\mathbf{r}) = \left[\frac{1}{q}\nabla M_M^J(q\mathbf{r})\right] \cdot \sigma, \\
T_5^{JM} &\equiv \Delta_M^J(q\mathbf{r}) = \mathbf{M}_M^{JJ}(q\mathbf{r}) \cdot \frac{1}{q}\nabla, \\
T_6^{JM} &\equiv \Delta'^J_M(q\mathbf{r}) = -i\left[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})\right] \cdot \frac{1}{q}\nabla, \\
T_7^{JM} &\equiv \Omega_M^J(q\mathbf{r}) = M_M^J(q\mathbf{r})\sigma \cdot \frac{1}{q}\nabla.
\end{aligned}
$$

T.S. Kosmas, PPNP 48(2002) 307 ; *Chasioti, Kosmas,* NPA 829 (2009) 234]

Single-particle reduced ME

One needs to compute the *single-particle reduced transition matrix elements* in order to perform explicit ν-nucleus cross sections calculations

$$\langle j_1 || T^J || j_2 \rangle = e^{-y} y^{\beta/2} \sum_{\mu=0}^{n_{\max}} \mathcal{P}^J_\mu y^\mu$$

with

$$y = \frac{q^2 b^2}{4}$$

$$n_{max} = (N_1 + N_2 - \beta)/2$$

For any semi-leptonic process in nuclei

$$\frac{d^2\sigma_{i\to f}}{d\Omega d\omega} \propto \left| \langle f || T_i^{J,M} || i \rangle \right|^2$$

Operator	β	$\mathcal{P}^J_\mu, 0 \leqslant \mu \leqslant n_{\max}$
$T_1^J = M^J = \delta_{LJ} j_L(\rho) Y_M^L(\hat{r})$	J	$E^J_{1,\mu} = (l_1 J l_2) \mathcal{U}^J_{J S_1} \varepsilon^J_\mu (n_1 l_1 n_2 l_2)$
$T_2^J = \Sigma^J = \mathbf{M}_M^{JJ} \cdot \sigma$	J	$E^J_{2,\mu} = (l_1 J l_2) \mathcal{U}^J_{J S_2} \varepsilon^J_\mu (n_1 l_1 n_2 l_2)$
$T_3^J = \Sigma'^J = -i[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})] \cdot \sigma$	$J-1$	$(J+1)^{1/2} E^{J-1}_{2,\mu} - J^{1/2} E^{J+1}_{2,\mu-1}$
$T_4^J = \Sigma''^J = [\frac{1}{q}\nabla M_M^J(q\mathbf{r})] \cdot \sigma$	$J-1$	$J^{1/2} E^{J-1}_{2,\mu} + (J+1)^{1/2} E^{J+1}_{2,\mu-1}$
$T_5^J = \Delta^J = \mathbf{M}_M^{JJ}(q\mathbf{r}) \cdot \frac{1}{q}\nabla$	$J-1$	$E^L_{3,\mu} = \mathcal{A}^-_L \zeta^-_\mu(L) + \mathcal{A}^+_L \zeta^+_\mu(L)$
$T_6^J = \Delta'^J = -i[\frac{1}{q}\nabla \times \mathbf{M}_M^{JJ}(q\mathbf{r})]$	$J-2$	$(J+1)^{1/2} E^{J-1}_{3,\mu} - J^{1/2} E^{J+1}_{3,\mu-1}$
$T_7^J = \Omega^J = M_M^J(q\mathbf{r}) \sigma \cdot \frac{1}{q}\nabla$	J	$E^J_{4,\mu} = \mathcal{B}^-_L \zeta^-_\mu(L) + \mathcal{B}^+_L \zeta^+_\mu(L)$
$\Omega'^J = \Omega^J_M + \frac{1}{2}\Sigma''^J_M$	$J-1$	$E^J_{4,\mu} + \frac{1}{2}\{J^{1/2} E^{J-1}_{2,\mu} + (J+1)^{1/2} E^{J+1}_{2,\mu-1}\}$

T.S. Kosmas, PPNP 48(2002) 307 ; *Chasioti, Kosmas,* NPA 829 (2009) 234]

Evaluation of the reduced ME

The single-particle reduced matrix elements have been evaluated by constructing a Mathematica code

evaluation of M.E

$$\langle j_1 || \Omega^J || j_2 \rangle$$

Major harmonic oscillator shells N

$$|n(l1/2)j\rangle$$

$\mathcal{P}^J_\mu$

Coefficients

Simple rational numbers

0f-0f

N'=3 N=3

1)	μ=0	J=1	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E0=-9\sqrt{\frac{2}{7}}$
2)	μ=1	J=1	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E1=\frac{54\sqrt{\frac{2}{7}}}{5}$
3)	μ=2	J=1	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E2=-\frac{108\sqrt{\frac{2}{7}}}{35}$
1)	μ=0	J=3	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E0=\frac{66\sqrt{2}}{35}$
2)	μ=1	J=3	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E1=-\frac{88\sqrt{2}}{105}$
1)	μ=0	J=5	$j1=\frac{5}{2}$	$j2=\frac{7}{2}$	n1=0	l1=3	n2=0	l2=3	$E0=-\frac{52\sqrt{\frac{2}{35}}}{21}$

Neutrino NSI at quark level

NSI of neutrinos with *d* and *u* quarks is described by:

$$\mathcal{L}^{NSI}_{\nu Hadron} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha,\beta=e,\mu,\tau}} \left[\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5)\nu_\beta\right] \left(\varepsilon^{qL}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1-\gamma^5)q\right] + \varepsilon^{qR}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1+\gamma^5)q\right]\right)$$

$\varepsilon^{qP}_{\alpha\beta}$ **Flavour changing contributions**

$\varepsilon^{qP}_{\alpha\alpha}$ **Non-universal terms**

with $(q = u, d \text{ and } P = L, R)$

and $L = \frac{1-\gamma_5}{2}$ $R = \frac{1+\gamma_5}{2}$

[*Barranco, Miranda and Rashba*, JHEP 12 (2005) 021]

Cross-sections calculations

Non-standard neutrino-nucleus differential cross section

$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{2\pi}\left[(G_V + G_A)^2 + (G_V - G_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 - (G_V^2 - G_A^2)\frac{MT}{E_\nu^2}\right]$$

T **: recoil nucleus energy**

For the coherent channel $G_A \approx 0$

with

$$G_V = \left\{\left(g_V^p + 2\epsilon_{ee}^{uV} + \epsilon_{ee}^{dV}\right) Z + \left(g_V^n + \epsilon_{ee}^{uV} + 2\epsilon_{ee}^{dV}\right) N + \sum_{\alpha=\mu,\tau}\left[\left(2\epsilon_{\alpha e}^{uV} + \epsilon_{\alpha e}^{dV}\right) Z + \left(\epsilon_{\alpha e}^{uV} + 2\epsilon_{\alpha e}^{dV}\right) N\right]\right\} F_{\mathrm{nucl}}^V(Q^2)$$

[*Barranco, Miranda and Rashba*, JHEP 12 (2005) 021]

The SM vector and axial vector couplings of neutrinos with protons and neutrons are

(ν, p) $\quad g_V^p = \frac{1}{2} - 2sin^2\theta_W \quad g_A^p = \frac{1.27}{2}$

(ν, n) $\quad g_V^n = -\frac{1}{2} \quad g_A^n = -\frac{1.27}{2}$

[*Giomataris*, Vergados Phys. Lett. 364 (2006) 23]

Outlook

- **Derive analytic expressions for the coherent and incoherent cross sections and perform QRPA calculations**
- **Compute branching ratios and determine upper limits on seesaw model LFV-parameters**
- **Exploit the μ-e conversion experimental sensitivity on the upper limits on the branching ratio and put limits to FCNC neutrino nucleus parameters**
- **Study the impact of FCNC processes to Astrophysics**

Thank you for your attention